Mathematics in the initial pre-service education of primary school teachers in Portugal: analysis of Gabriel Gonçalves proposal for the concept of unit and its application in solving problems with decimals

Rui Candeias

Agrupamento de Escolas Terras de Larus, UIED/FCT Portugal; rp.candeias@campus.fct.unl.pt

This study aims to analyze Gabriel Gonçalves didactic proposal for the initial approach of rational numbers in primary education, published in the 1974 edition of Didactic of Calculation (Didática do Cálculo). At a time where modern mathematical ideas began to influence mathematics education in the early years, it is important to understand how mathematics was addressed in the pre-service education of teachers of this school level. Among the themes approached in this text book the choice of rational numbers was due to the many difficulties that primary students usually have, as well as the teachers in teaching them. The study was conducted in a historical perspective based on documents analysis. The proposal of Gabriel Gonçalves emphasizes the initiation of rational numbers through decimals instead of fractions and the different types of problems that should be presented to students in the initiation to decimals.

- *Keywords: History of mathematics education; Elementary school mathematics; decimals*

Introduction

This paper is part of a broader work that aims to characterize, in a historical perspective, mathematics in the initial pre-service education of primary school teachers in Portugal, either from the point of view of the centrally issued legislation or the analysis of textbooks for teachers, which constitute a point of view closer to the practices. This is a pioneering study that will focus the analysis of the teacher training textbooks on their approach to non-negative rational numbers. It is intended to select a set of representative authors from different periods, between 1844 and 1986, and examine how the proposed initiation to non-negative rational numbers in the early years of schooling evolved. In this article, I selected a representative author of the beginning of the 1970s, whose proposal of approach to rational numbers will be described.

This paper analyzes the didactic proposal of Gabriel Goncalves, former teacher of the Primary Teacher Training School of Porto (Escola do Magistério Primário do Porto), for the initial approach to rational numbers presented in the textbook for teachers of the Special Didactic (Didática Especial) discipline of initial pre-service education of primary school teachers, entitled Didactic of Calculation, 1974. The main questions in this paper are: What is the initial proposal for the teaching of non-negative rational numbers presented by this author? What kind of representations are privileged? What kind of teaching materials are referred to? What importance does this author give to the definition of unity? What are the most used contexts for the display of decimal numbers? What kind of problems does the author propose to students?

According to Chartier (1990), the pedagogy textbooks for teachers constitute a source for the history of teacher professionalization, and are a sample of what constitutes teachers professional knowledge. Pintassilgo (2006) considers these textbooks for teachers a major instrument of innovation and control, legitimizing certain ideas and practices, and simultaneously withdrawing this legitimacy to others. Moreover they are important resources in building a school culture and as guides in classroom and students management, as well as in the professional teacher development.

The importance of research in the context of the history of mathematics teaching is not limited to the knowledge of the past. Chervel (1990) points out that, through the historical observation, we can bring this disciplinary models and operating rules whose knowledge and exploitation may be useful in discussions about teaching today. In this sense, Matos (2007) states that knowledge of the past may allow an action more grounded in the present. In this perspective, it is important to see how it was done the training of primary school teachers, in a mathematical topic as the rational numbers, at a moment when the Modern Mathematics Movement begins to emerge, favorable to an active construction of knowledge and the use of structured materials in mathematics teaching. The operationalization of the analysis of the work of this author was conducted as an historical study based on the collection, selection of sources and documentary analysis, as defined by McCullough (2004).

Pre-service education of primary school teachers during the New State regime (Estado Novo)

In Portugal, the military dictatorship implanted in 1926, and later the New State regime that followed, changed the pre-service education of primary school teachers (Pintassilgo, 2012). In 1930, still in the transition from military dictatorship to the New State regime, the Normal Primary Schools (Escolas Normais Primárias) were replaced by Primary Teachers Training Schools (Escolas do Magistério Primário) involving a radical change in school organization, the curriculum framework and, later on the syllabi, in 1943.

With the restructuring of the course in 1942, and the syllabi published in 1943, the mathematical content of the programs of the Primary Teachers Training Schools came to be centered on teaching and methodological dimensions of the primary content. In these 1943 syllabi, did not exist any discipline with mathematical content. The reformulation of syllabi in 1960 reinforced the discussion on teaching methodologies. These syllabi had no disciplines with mathematical scientific content, situation that remained until 1975 syllabi. It is in the context of these 1960 syllabi that the Didactic of Calculation, analyzed in this paper, was published.

Decimal numbers in the teachers textbook of pre-service education of primary school teachers: Gabriel Gonçalves approach in *Didática do Cálculo*

Didatics of Calculation (Didática do Cálculo), composed of two volumes published in 1972 and in 1974 by Porto Editora, is part of a set of textbooks for teachers written from the 1960s to serve as a support to the discipline of Special Didactics B of the courses of pre-service education of primary school teachers. According to the author, Gabriel Gonçalves, former professor of the Primary Teachers Training School of Porto and inspector-advisor at the time of the edition of the manual, Didactics of Calculation was mainly intended for students-masters of the Primary Teachers Training Schools although it could also be used by all of those who were interested in education issues.

From chapter VII to chapter XIII of the second volume of Didactic of Calculation, Gonçalves (1974) addresses the teaching of decimals. Due to space limitations, this article will not address the entirety of this author's proposal. This paper focuses on chapter VII which deals with general aspects of the teaching of decimals and chapter XIII dedicated to the concept of unit and its application to problems, which are important aspects of the teaching of these numbers.

The chapter VII, entitled "Preparation of the study of decimals; measurements with linear units already known. Writing and reading of representative numbers of these measurements; using the decimal point" is organized into three main sections: 1) Goals; 2) General considerations and 3) Direction of Learning. In the goals, the author begins by pointing out that the aim is that the child expands his knowledge of decimal number system, extending it to tenths, hundredths and thousandths. These concepts would appear as an extension of the base ten numbering system.

Gonçalves (1974) presents subsequently some general considerations about the teaching of decimals and fractions, starting by putting the question if the teaching of rational numbers should be done with decimals instead of common fractions. On this issue, Gonçalves (1974) presents two opposing trends. On the one hand, quotes methodologists[1] that, according to Gonçalves (1974), claim that one should start with decimal fractions in its decimal representation, because it would be "as a continuation of the study of decimal number system, but with numbers lower than the unit "(p. 38). He presents the submultiples of length measurements, capacity and weight, as examples stating that:

> Each of these units contains ten units of the next lower order. So, we can operate in the written calculation as if they were natural numbers. And as the calculation with decimal is much easier than with fractions, it will be with the decimal fractions, in the form of decimal representation, which should start. "(Gonçalves, 1974, p. 38).

On the other hand, he presents the opinion of methodologists[2] who claim that we should begin the study by the common fractions, of which the decimal fractions would be only a case. Then he forwards the arguments of these authors, stating that:

> The calculation becomes more intuitive and rational: the half, third, fourth, ..., are easier to understand than the tenth, the hundredth, ... The calculation of common fractions prepare better for the decimal than the contrary. (Gonçalves, 1974, p. 38).

Given these two divergent trends, Gonçalves (1974) refers that he will follow the first, as it was prescribed in primary syllabi at the time[3] that is, starting with decimals representation. In section 3 of this chapter, called the Direction of learning, Gonçalves (1974) shows what stood as the teaching of decimals in the primary education syllabi of the time. This topic was considered "the greatest obstacle to overcome in the 3rd grade" (p. 39). According to the primary syllabi, the approach to

[1] On this subject, Gonçalves (1974) quotes methodologists like Büttner, Tank or Pikel, but does not identify the works of reference of these authors.

[2] On this subject, Gonçalves (1974) quotes methodologists like Böhme or Hentshel, but does not identify the works of reference of these authors.

[3] At the time were in effect the syllabi approved in Decree No. 23,485, Government Daily, 167, 16.07.1968, 1019-36.

decimals should be made from the length measurements, placing students in situations that were necessary to measure with meter and decimeter. These measurements would express numbers in what was called mixed decimals, numbers with a whole part, that after a decimal point had a decimal part. After working with these mixed decimals, students should verify that the rules used with whole numbers also applied to decimal numbers, "the numbers continue to have an absolute value and a position value." (p. 39). After performing this work, situations that could lead them from mixed decimals for simple decimal numbers should be offered to students.

Gonçalves (1974) establishes a relationship between the perspective in the previous two chapters of his manual, which addressed the metric system, with measurements only with positive integers, in the final part of his general considerations. In this chapter, he proposes to address measurements, using the decimal notation, with the decimal point.

Gonçalves (1974) continues the chapter with section 3. Direction of Learning, with the suggestion of some techniques and activities for the introduction of the concept of the tenth, starting from decimeter, and the notion of the hundredth and thousandth, from notions of centimeter and millimeter. The author proposes the introduction of the tenth in eleven steps (table 1):

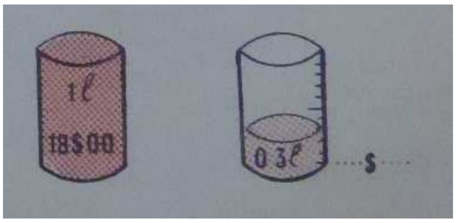
1) Measurements, in which the meter is used a whole number of times;	2) Measurement expressed a whole number of times in meters and decimeters (ex .: the picture measures 1 m and 2 dm);
3) In the measurements, identification of the entire unit, the meter, and the tenth of the entire unit, the decimeter. Representation in conventional manner, with the decimal point and the identification of the position value.	4) Identification that the rule that governed the whole numbers also apply in decimal numbers: "In a number, any digit at the right of other is order units ten times smaller than the first one" (p. 30);
5) Measurements that result in mixed decimal representation. Registration in tables;	6) Measurements that result just with decimal part. Lead students to understand that the zero to the left of the decimal point is the absence of whole units;
7) Exercise that does not exceed the unit Ex.: 3 dm + 2 dm = 5 dm 0,3 m + 0,2 m = 0,5 m	8) Exercises which form exactly the unit Ex.: 5 dm + 5 dm = 10 dm 0,5 + 0,5 m = 1,0 m = 1 m
9) Exercises that exceed the unit Ex.: 4 dm + 5 dm + 3 dm = 12 dm 0,4 m + 0,5 m + 0,3 m = 1,2 m = 1 m + 0,2 m	10) Exercises Ex.: 0,4 m = 0,1 m + 0,1 m + 0,1 m + 0,1 m = 0,2 m + 0,2 m = = 0,3 m + 0,1 m
11) Application and verification exercises.	

Table 1. Proposal for an approach to decimal (Gonçalves, 1974)

Chapter XIII, titled the "Expansion of the Unit Concept. Its application to solve problems with decimals" is divided into two sections: 1. Goals and general considerations and 2. Preparatory exercises. In a footnote at the beginning of this chapter, Gonçalves (1974) draws attention to the possible application in problems with common fractions[4]. The first section provides some general considerations about the unit and its nature. Gonçalves (1974) begins by distinguishing the single units of the 1st or 2nd order units, such as ten or hundred, or units designated as decimals, as 0.1; 0.01. Also distinguishes other composite units as the dozen or the quarter of a hundred or other sets as a basket of oranges that can be considered as a whole. For Gonçalves (1974), this expansion of the concept of unit "is the basis of an important branch, allowing you to easily solve questions that otherwise would be too complex" (p. 79) and therefore should be developed in children. Gonçalves (1974) points out that many problems with decimals have the following expressions: "the amount corresponding to the unit; the fraction[5]; the amount corresponding to that fraction or else their counterparts, the value of the unit (the whole); the fraction; the value of the part corresponding to the fraction "(p. 79). He points out that being given two of the above items is always possible to find the third, and stresses that this implies the possibility of formulating three groups of problems: 1) given the amount corresponding to the whole and the fraction, find the value of the part corresponding to that fraction; 2) given the fraction and the amount corresponding to that fraction, find the value of the whole; 3) given the value of the whole and the value of a part of the unit, find the fraction which corresponds to that part.

The author presented several examples considered similar, for the first group of problems. The first problem, with a context of capacity measures, is to find the amount corresponding to the respective unit. For this type of problem is presented a resolution, first find the value of the decimal unit 0.1, and then multiplying by the number of times it is repeated, in this case, multiplying by three.

> 1) Each liter of olive oil costs 18$00. How much will cost 0.3l of that olive oil?
>
> The problem can be solved, first finding the decimal value of each unit (18$00:10 =1$80) and then multiplying it by number of decimal units (1$80x3= 5$40). (Gonçalves, 1974, p. 79)



Soon afterwards a second example is shown. It is also an iterative situation that leads to the meaning of multiplication and can be considered a counterpart of the first: "2) Each liter of olive oil costs 18$00. How much will cost 3 l of the same olive oil "(p. 80). The resolution is the multiplication of 18$00 by 3, 18$00x3=54$00. Gonçalves (1974) believes that after children observe the resolution of the second problem they will eventually realize that the action in the first problem is also multiplicative, noting that multiplying by 0.3 is the same as dividing by 10 and

---

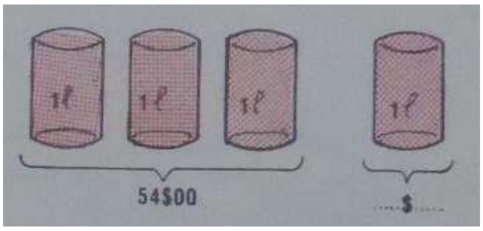
[4] The footnote with reference to common fractions is placed in the text by the author, because in later chapters, when it comes to addressing the common fractions, will present the same kind of problems. However, we will not address common fractions in this paper.

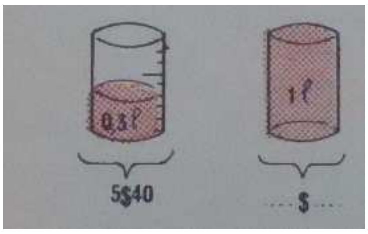
[5] Gonçalves (1974) uses the term "fraction", in the sense of part of a whole and not in the sense of fraction representation of the rational number.

multiply the result by 3. He points out that the child will also conclude that" *given the value of the unit*, *to know the amount* (whether higher or lower than the unit), *the action is multiplicative*."(p. 80, italics in original). For this first group of problems are still presented other examples. According to Gonçalves (1974), the intention is to extend the concept of unit, to the concept of the whole.

In the second group of problems titled "given the fraction and the amount corresponding to that fraction, find the value of the whole" several problems are presented. The first two are in the context of capacity measures and the author intended that should be solved by analogy.

1) They bought three liters of olive oil for 54$00. How much did cost one liter?





2) Were purchased three deciliters (0.3 l) of olive oil for 5$40. How much did cost one liter? The meaning of the first problem is clearly partitive (54$00:3=18$00). (Gonçalves, 1974, p. 80)

The first problem is associated with a partitive situation and the solution involves a division, 54$00:3=18$00. From this, the student should recognize that the second problem presents a similar situation, inferring that if you know the value of a certain amount, to know the unit, the meaning is to divide. For the second problem another kind of solution is suggested: first determine the price of each tenth and then multiplying this result by 3. Other examples of similar problems are then presented.

In the third group of problems, "given the value of the whole and the value of a part of the unit, find the fraction which corresponds to that part", are initially presented two problems.

1) With 54$00, which portion of olive oil can we buy, whose price is 18$00 per liter?

2) With 5$40, which portion of olive oil can we buy, whose price is 18$00 per liter? (Gonçalves, 1974, p. 82)

The first problem is considered to be the division quotative meaning, and the resolution proposed is 54$00:18$00=3. The second problem is also framed in a similar reasoning and therefore should be solved similarly 5$40: 18$00 = 0.3. To Gonçalves (1974), these two problems comprise the quotative meaning. Gonçalves (1974) points out that "*the fraction is given to us by the relationship (or ratio) between the value of the quantity and value of the unit*." (p. 82, italics in original). It points out that learning these problems should not only be supported by the memorization of rules, or repetition, without be a prior understanding. Gonçalves (1974) considers that this understanding of work was previously done when they were worked multiplication and division of decimals as a generalization of the basic rules of these operations with whole numbers.

In section two of this chapter, entitled preparatory exercises, are suggested three different types of problems, 1. Recognize (or find) the fraction; 2. Find the value of the fraction; 3. Find the value of the unit (or the whole) which corresponds to the sorts of problems previously shown. Gonçalves (1974) begins by highlighting that for the understanding of the basics for learning problem solving, should be practiced some sensory exercises, called concrete phase, such as manipulation, paper folding, drawing, of which the preparatory exercises were examples.

Concluding remarks

In chapter VII of the manual in analysis, the first dedicated to the teaching of decimals, Gonçalves (1974) starts by discussing where to begin the study of rational numbers, by the decimals or by fractions. In his work, Gonçalves (1974) follows the indication of the primary school official syllabi of that time and primarily addresses the rational numbers by its decimal representation. Gonçalves (1974) also refers to arguments of different authors. Regarding the teaching of rational numbers, Brousseau, Brousseau and Warfield (2007) also present a discussion on the best way to introduce them to students considering that it is not necessary to know fractions to learn decimals. Rather, decimals can be understood at once as a decimal number, supported by the decimal measuring system, allowing that all practical measurement problems can be solved more easily. They consider that this solution has many advantages for teaching, especially in countries where children are already familiar to the use of metric measurements.

The different syllabi of mathematics discipline for primary education in Portugal also seem to reflect this discussion. In the 1960s two syllabi to this level were in force. In both cases the rational numbers were discussed in the third grade from decimal numerals, with the use of linear measurements. The fractions were worked only in the fourth grade, but only worked the concept of fraction. In the syllabi for the school year 1974/1975 the introduction of rational numbers was still made with decimal representation and working with fractions was no longer part of the primary syllabi, happening the same in 1975 syllabi. However, in the 1978 syllabi, the chapter devoted to rational numbers deals first with the fractions and then the decimal representation. In the 1980 syllabi, rational numbers were again addressed exclusively by decimal representation. In 1990, the official syllabi for primary education began the work with rational numbers in the second grade, with the fractions, but only had an applied operator to a discrete set. Afterwards and until 5th grade rational numbers were worked out just with decimal representation.

Gonçalves (1974) distinguishes different types of units, referred to as units of "various kinds". He defines the single unit, but ten and hundred are first and second order units, that means they are composed units. Other composite units are also presented, called set-unit as the dozen or a quarter of a hundred. Monteiro and Pinto (2009) highlighted the different types of unit as one of a major difficulty in the study of fractions in the early years.

This unit concept is considered by Gonçalves (1974) as essential for solving problems with decimals, because of that definition results the possibility of grouping the problems into three distinct groups: 1) find the value corresponding to a part of the designated fraction; 2) find the value of the whole giving a part; 3) find the part of a whole.

In the presenting of the problems, Gonçalves (1974) emphasizes the symbolic representation, but also presents some problems, and the respective resolutions, illustrated with pictorial models. However, in the second section of chapter XIII, he highlights the importance of concrete phase, suggesting the use of sensory exercises using the manipulation, paper folding and drawing. However, no structured didactic materials are referred.

Synthesizing, in this proposal of Gonçalves the importance attributed to the work with the decimals stands out by the affinity with the calculation with the natural numbers that the students previously worked. This option is based on the curriculum that was in force in Portugal at the time, but methodologists from other countries who support this option were also mentioned. As the works of the mentioned methodologists are not mentioned, it is not possible at this moment to attest to the influence they had on the proposal of this portuguese author. This is a discussion that continues today as it is possible to see in the works of Brousseau et al. (2007) and the successive changes in the approach to rational numbers presented in the portuguese curricula.

The work with different units and the importance given to the definition of the unit it is also relevant in Gonçalves work. This definition of unit allows Gonçalves to systematize the type of problems to be presented to the students in the beginning of the learning of this numerical set. The most used contexts in these problems are those of measurement (length, weight).

References

Brousseau, G., Brousseau, N. & Warfield, V. (2007). Rationals and decimals as required in the school curriculum: part 2 - from rationals to decimals. *The Journal of Mathematical Behavior*, *26*, 281-300. http://dx.doi.org/10.1016/j.jmathb.2007.09.001

Chartier, R. (1990). *A história cultural: entre práticas e representações*. Rio de Janeiro: Difel.

Chervel, A. (1990). História das disciplinas escolares: reflexões sobre um campo de pesquisa. *Teoria & educação*, 2, 177-229.

Gonçalves, G. (1972). *Didáctica do Cálculo 1.º volume*. Porto: Porto Editora.

Gonçalves, G. (1974). *Didáctica do Cálculo 2.º volume*. Porto: Porto Editora.

Lamon, S. (2006). *Teaching fractions and ratios for understanding*. London: Lawrence Erlbaum Asssociates Publishers.

Matos, J. (2007). História do ensino da matemática em Portugal — a constituição de um campo de investigação. Em J. M. Matos e W. R. Valente (Eds.), *A Matemática Moderna nas escolas do Brasil e de Portugal: primeiros estudos* (pp. 8-20). São Paulo: GHEMAT.

McCulloch, G. (2004). *Documentary Research: In Education, History and the Social Sciences.* London: RoutledgeFalmer.

Monteiro, C., & Pinto, H (2009). *Desenvolvendo o sentido do número racional*. (2.ª ed.). Lisboa: Associação de Professores de Matemática.

Pintassilgo, J. (2006). Os manuais de pedagogia no primeiro terço do século XX: entre a tradição e a inovação. Em Pintassilgo, Joaquim et al. *História da escola em Portugal e no Brasil: circulação e apropriação de modelos culturais*. (pp. 175-200). Lisboa: Edições Colibri.

Pintassilgo, J. (Coord.) (2012). *Escolas de formação de professores em Portugal*. Lisboa: Edições Colibri.