Theoretical constructs for early intervention programs in mathematics: who cares? – A Danish example

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"There is nothing so practical as a good theory". The statement from Kurt Lewin (1945) is frequently cited, also in mathematics education. The statement invites for and requires close cooperation between different agents, whatever their own specific relation to practice and theory is. It is not a straightforward endeavour. One reason is that the term theory as well as the term practice may very well hold different meanings by different agents. This paper explores perceptions of theory held by different agents, focusing on perceptions held by mathematics teachers (with their inherent interest in practice). We draw on experiences and results from projects on early intervention in mathematics education in Denmark. The paper exemplifies how agents' quite different work conditions and requirements seem to constitute qualitative different needs for theoretical constructs. In our view such exploration are not fully recognized in the usual paradigm of "mathematics education research", but can be a valuable element in "implementation research". Our exploration leads us to expand Lewin's statement with two questions "Who cares for a good theory?" and "What makes a good theory good for whom?"

Keywords: Early intervention programs, teaching principles, theoretical constructs.

Background for the early mathematics intervention project

We notice a long tradition for integrating programs for early mathematics intervention into compulsory education practice like e.g. Mathematics Recovery (MR) and Extending Mathematical Understanding (EMU) Intervention Program, in the countries Australia, the UK, Ireland and the USA. We notice a similar tradition in Denmark for programs for early reading intervention as such programs are implemented at a regular basis in many schools in Denmark. Either as part of a municipal policy or a matter of choice, schools launched early (from the first grade) intervention processes in reading to support individual students, who show signs of reading difficulties. But no intentions of integrating programs for early mathematics intervention into compulsory education practice were seen in Denmark until about a decade ago.

Since 2007 we notice a growing concern from a diversity of agents in Denmark for supporting students in mathematical difficulties: teacher educators and researchers in mathematics education, educational authorities, politicians, teachers and private funds. Teacher educators and researchers in mathematics education started to aim for developing and communicating theoretical knowledge and experiences of teaching principles for early intervention in mathematics. School mathematics teachers started to aim for effective programs with inspiring and comprehensible framework, teaching principles and materials. These are the agents that this paper focuses on. Other agents, who

also may hold alternative perceptions of what is theory, count educational authorities, politicians and private funds: At the national level concerns were raised about students failing at mathematics as early as in the official guidelines to the 2003 national mathematics curriculum from the Ministry of Education (UVM, 2003). The official guidelines to the revised 2009 national curriculum (UVM, 2009) for the first time described the issues in detail. Back in 2004 an OECD Review emphasized the need in Denmark to support failing students in mathematics in the first school years (Mortimore et al., 2004). Educational authorities at national and municipality levels in Denmark then started to aim for implementation of programs with clearly stated effects. When politicians' time horizon ends at the next Election Day, their concern aims towards quickly shown intervention effects, although educational policy in Denmark has a custom of being decided as broad democratic compromises. Finally private funds started showing interest in early mathematics intervention programs, and it seemed that they seemed to aim for evidence results based on RCT-experiments (randomized controlled trial).

Our practical involvement in early mathematics interventions began in 2007, when local politicians and school authorities at the municipality Frederiksberg (in the Copenhagen area) decided to prioritise developmental projects for mathematics in their 9 public schools in the period 2007-2013. The first Danish piloted program for early intervention in mathematics is literally called Early Mathematics intervention at Frederiksberg. The Danish abbreviation is TMF (in Danish: Tidlig Matematikindsats Frederiksberg).

The pilot project was conducted by Peter Weng, Metropolitan University College and Lena Lindenskov, DPU. It was financed by Frederiksberg Municipality, Metropolitan University College and DPU. It was an educational design research, as it followed the generic definition of Barab and Squire (2004):

> A series of approaches, with the intent of producing new theories, artefacts, and practices that account for and potentially impact learning and teaching in naturalistic settings.

The pilot project – and the couple of follow-up projects – are all examples of "implementation research", explicitly based on and informed by findings from mathematics education research.

Perceptions of what is good theory – by mathematics school teachers

The following exploration on perceptions of what is good theory in the sense of Lewin, held by the involved mathematics teachers, draws on our interactions with the teachers in the many phases of the pilot project from

Interactions happened at the seminars[1], training sessions, coaching sessions, and by e-mails. The minutes from seminars were recorded.

[1] Mathematics Recovery Programme (MR) was a source of inspiration for the pilot project (Wright et al., 2006; Ellemor-Collins & Wright 2007). This is why a teacher seminar was arranged with Ms. Noreen O'Loughlin from Mary Immaculate College, University of Limerick about hypotheses and concerns in Mathematics Recovery and in the Frederiksberg pilot.

The specific choice of mathematical and other aspects for the framework and material for the early intervention was made in a dialectic process with the 16 pilot mathematics school teachers at Frederiksberg, chosen by their school principals, which is also described in Lindenskov & Weng, 2014.

The framework and material were meant to guide, frame and inspire the teachers. The teachers could actually use the material directly together with their students, but the teachers were encouraged to adapt it to the individual student's need and motivation.

The draft framework and draft materials were developed through four developmental cycles (1) – (4). From January 2009 to September 2009, Weng and Lindenskov developed First draft material, based on international mathematics education theory and Danish empirical knowledge of mathematics in life and of primary and lower secondary school mathematics. Weng and Lindenskov initially doubted whether the teachers would find it relevant to study justification and theory underpinning the choice of mathematical areas, evaluating and teaching principles, and materials. That is why, only few theoretical constructs and justifications were in the first draft communicated to the teachers. *But,* as the structure and each part of the draft material were critically explored and discussed during the week long teacher training sessions 14 – 18 September 2009, this perception of the teachers' perceptions of their needs for theory showed to be wrong. The teachers endorsed the underlying ideas, but actually asked for further explanation of rationales and theoretical constructs. The teachers also asked for an extensive introduction to the program as such. The teachers had no time provided for reading research articles by themselves, so the teachers got theory and justifications presented orally with power point slides at every morning session during the training week. Finally the teachers suggested expanding of the mathematical content, including measurement as a mathematical area in itself and its use of measurements in other mathematical areas.

(2) Lindenskov and Weng developed Second draft of material based on the pilot teacher feedback and feedback from research assistant, Tina Kjær. The draft included justification and theoretical constructs underpinning more of the mathematics areas Second draft was sent to each school October 2009 for experimenting. Each pilot teacher tried out specific parts of the material in the fall of 2009. The distribution of the parts to each school was decided through discussions among all pilot teachers. Each pilot teacher was requested to try out two or three activities with as many students as possible. The age of the students was not important. If possible, more material was to be tried out. The pilot teachers were given a specific task in order to evaluate the materials: they were asked to document in as much detail as possible - by writing in premade tables - how each mathematics task and each mathematics and attitude question led to student-teacher conversations which could indicate the student's thinking. The experiments were concluded with a seminar on 3 December, where each pilot teacher presented results. Anything that had particularly surprised the

teachers was also presented, and some common concerns were then discussed. The teachers put forward that the newly added justifications and theoretical constructs were helpful. A teacher said: *or else we do not know why the chosen mathematical areas, concepts and competences are important to focus on*.

(3) Based on these results Weng and Lindenskov developed Third draft material and introduced it at a seminar on 28 January 2010, and justifications and theory were included for all mathematical areas. The following months, each pilot teacher tried out parts of the material with a number of students. This time all the students were in the second grade. The aim was to allow the pilot teachers to experience the structure of the material and to practice student-teacher conversation. Peter Weng visited and coached every teacher once, and the teachers were invited for mail correspondence at any time during the pilot study.

At a midway seminar on 9 March, the teachers described their general impression as positive, and the material was generally considered adequate by the teachers. Several pilot teachers said they found it motivating to work with the material together with the students, and that they had heard from the students' ordinary mathematics teachers that the intervention seemed to have a positive impact on the students' learning process.

Issues regarding the scope and range of the material were discussed, for instance how to prioritize between broad presentations of many mathematical aspects or assuring narrow success in fewer mathematical areas. The risk that the material put severe strain on teachers, especially when they were unfamiliar with it, was also discussed. To illustrate this discussion, we have listed three teacher transcripts and one teacher trainer (Weng) transcript below:

Teacher 1: I feel pinned down by the material. I feel like, 'Now I must do this, then I must do that,' and you have to look for concrete material yourself. It is very restraining. While I look for extra material, I give the students small tasks on the computer to work with.es, OK.

Teacher 2: The material could be constraining. But the material is important as a database of ideas. The material gives me ideas. It supports my own inspiration process, and it helps me to include everything in my practice.

Teacher 3: The material is useful, when I prepare the intervention sessions.

Weng: Try to think about the material as something that provides you with opportunities and inspiration. We invite you to a flexible adaptation to specific students.

(According to the minutes, our translation)

The final seminar on 27 May 2010 discussed organisational and psychological issues in detail. Additionally the seminar concluded with a number of specific suggestions for the upcoming editing process:

- Include more material on geometry, i.e. two modules on forms and figures instead of just one

- Include more questions and activities about weight and explain relations between measuring and other aspects
- Compile a list of materials and recommended internet sites
- Adopt an easy to read typography
- Place the material in binders
- Split each page into parts in order to make it easy to get an overview
- Reorganise the pages in order to make it easy for the teacher to connect assessment activities and instructional activities.
- Include more supplementary materials, to be used in student interviews on beliefs, attitudes and feelings towards mathematics: i.e. cards with words or pictures that the students can choose from and that might inspire them. (Dowker, 2004)
- Recommend to organise these interviews during the whole intervention period, not just as a starter.

(According to the minutes, our translation)

No more comments on the need for justification and theory were made.

(4) Until 12 August 2010 the fourth draft were developed, which was to be used from 2010 onwards in the regular TMF for individual second grade students in all of Frederiksberg's public schools. The research assistant, Tina Kjær, examined the material and ensured that the teachers' suggestions were taken into account.

Strongly supported by the pilot teachers' feedback some organizational and psychological aspects of individual students' learning and instruction were included as just as important as the mathematical aspects.

(5) The material went through further revisions in collaboration with a commercial publisher. In 2013 a book based on the material was published (Lindenskov & Weng, 2013) and translated into Norwegian. This book for primary school was in 2016 followed by a similar book for lower secondary school (Lindenskov, Tonnesen, Weng, 2016).

In order to exemplify the level of communicated theoretical constructs and justifications, the table below (translated by us) shows what was communicated in the 2013 book in relation to the mathematics area "Basic strategies for numbers in addition and subtraction".

Theoretical constructs and justifications	**Communicated in the 2013 book**
Relational understanding (RU) and instrumental understanding (IU) Although IU in its own context is often easier to understand and gives correct answers with less	When the student experiences a productive development in his/her basic strategies for numbers in addition and subtraction this area, it opens for the student's possibilities further on to

knowledge involved, RU is more adaptable to new tasks and easier to remember. Skemp, R. R. (1976/2006). Relational understanding and instrumental understanding. *Mathematics Teaching in the Middle School, 12*(2), 88–95. Originally published in Mathematics Teaching.	become capable in doing relevant addition and subtraction and to use it in many contexts. Also, potentially this experience will contribute to another highly relevant math competence: good estimating skills for big numbers.
Constructivist teachers' primary activity is communicating with students. In the constructivist view, teachers should continually make a conscious attempt to "see" both their own and the students' actions from the students' points of view. Cobb, P.; Steffe, L.P. (1983). The Constructivist Researcher as Teacher and Model Builder *Journal for Research in Mathematics Education, 14*, 83–94.	Some teachers may have a too simple perception of constructivism that may imply teachers should not interfere with students' own development of strategies because the pupils by themselves would develop at the pace most optimal. But we know, it is a risky affaire.
Students who engage in strategy development decisively perform better in the long run than students who don't. Ostad, Snorre (2008). Children With and Without Mathematics Difficulties. Aspects of Learner Characteristics in a Developmental Perspective, In: Ann Dowker (ed.). *Mathematical Difficulties: Psychology and Intervention*. Elsevier, 143 – 153. *Ostad, S.A. (2013).* Strategier, strategiudvikling og strategiundervisning med fokus på den fundamentale matematiklæring. I: M.W. Andersen and P.Weng (red.)*: Håndbog om matematik i grundskolen. Læring, undervisning og vejledning* (103-113).København: Dansk Psykologisk Forlag	Students who from early on start developing their strategies, tend to continuously improve existing strategies and increase the number of strategies. Contrary, students who from early on stick to their strategies, tend not to start improving them later on. It is shown that students, who stop developing their strategies, will have to toil hard and will still lack behind. (…) Do not just present materials for the students to acquire new further leaning. Let the student use materials and activities in order to consolidate what is almost or recently learnt as a means to improve the student's self-confidence and realistic perception of own addition and subtraction skills. We recommend that the teacher talk with the student about his/her strategies, e.i. by regularly ask how long time this strategy is used, if the strategy leads to right results, if the student use other strategies, too, or thinks other strategies could be used.

	Appropriate further learning may well be about strategy development.

Conclusion

During the development phases (1) – (4), the pilot teachers endorsed the underlying ideas of the intervention project, and asked for the rationale behind every included aspect to be explicitly communicated. They encouraged to give more extensive introduction and to expand the included measurement aspects into two measurement aspects.

The teachers explicitly endorsed the theoretical construct of "math-holes", as they said it helped them to acknowledge many opportunities to help the students and to identify students' potentials and motivation while exploring and developing their mathematical needs. The teachers appreciated that the material gave a firm frame and at the same time invited and inspired the teachers to adapt and further expand the materials to the specific learning situations with the students. The teachers recommended the material to be expanded with more mathematical concepts and competences, which are considered relevant in the Nordic contexts (i. e. Dalvang & Lunde, 2006; Niss & Højgaard, 2011), because the teachers found the competences potentially troublesome for the low achieving students. Furthermore the teachers suggested an expansion of the materials' concepts and strategies on measurements and part-whole.

For the mathematical teacher educator and the educational researcher it was a challenging task to choose theoretical constructs to underpin the intervention. The context had to be considered (see Lindenskov et al, 2016), Too, it was a challenging task to communicate the chosen theoretical constructs to the teachers. Genres from scientific journals were not appropriate. Instead it was communicated as justified practical advices.

We therefore conclude,

- that the teachers asked for further practical ideas and materials to be used directly or to be adapted to specific students' needs and motivations,
- that the teachers did not ask for more clarification of the theoretical constructs and justifications underpinning the program, than were communicated to them already,
- that theorists as well as practitioners care for theory, but in very different ways.

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