Preliminary testing of apps in mathematics, based on international collaborative work

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In this paper, we report on a project about developing mobile applications for learning mathematics through game playing. Several different types of applications were developed in a collaboration between universities in Norway and Slovakia, and between teacher education and information science. We give some preliminary results on how two of these applications were received and used by Slovakian pupils.

Keywords: mobile learning, game based apps, mathematics learning, collaborative work

Introduction

Mobile devices, such as smartphones, tablets and laptops, have become an integral part of our lives. Teachers and pupils use them daily for communication, searching for information or for entertainment. Pupils today, born from 1990 to 2010 and recognized as generation Z, are the most technologically advanced generation, often known as digital natives. They were born into the era of the Internet and Facebook; they always want to stay connected with their friends and to use high-speed digital devices (Baker & Evans, 2016). Hence, the wide spread of mobile devices causes a natural social pressure and challenge for educators to include these devices into education, to support learning. Using technology in the classroom is not a new idea. Computers, laptops, and netbooks have all been added to classroom settings with the hopes of revolutionizing education, promising vast improvements to pupil outcomes. These technologies, largely, have left education unchanged and in a continual state of need for improvement (McQuiggan, Kosturko, McQuiggan & Sabourin, 2015). Mobile learning offers a novel approach to reach current pupils. By the term *mobile learning* we follow McQuiggan, Kosturko, McQuiggan and Sabourin (2015, p. 31).

> It is anywhere, anytime learning enabled by instant, on-demand access to a personalized world filled with the tools and resources we prefer for creating our own knowledge, satisfying our curiosities, collaborating with others, and cultivating experiences otherwise unattainable. Mobile learning implies adapting and building upon the latest advances in mobile technology, redefining the responsibilities of teachers and students, and blurring the lines between formal and informal learning.

Mobile learning offers flexibility in when learning takes place, personalization of content, and gives pupils experience with contemporary technology and relevant skills for the future. So unsurprisingly, mobile learning has been considered as the future of learning or as an integral part of any other form of educational process in the future (Trifonova, 2003).

In June 2016, gaming apps were the most popular apps based on availability, as about 23 % of all apps available in the Apple App Store fit in this category. The second most popular category was Business (10.22 %), closely followed by the Education category (9.21 %) ('Most popular', 2016). Shuler (2012) has analyzed the Education category from Apple App store. In 2011, more than three quarters (77 %) of the top selling apps targeted preschool or elementary aged children. Early learning was by far the most popular subject/skill-set (47 %), followed by mathematics (13 %). Drigas and Pappas (2015) have analyzed the most representative studies of recent years (2002 - 2013), involving online and mobile applications and tools for mathematics as well as their effect in the educational process. The results of the studies revealed that online and mobile learning applications motivated pupils, making mathematics instruction more enjoyable and interactive than ordinary teaching practices. The analyzed applications were targeted towards one specific area of mathematics, like graphs and functions, arithmetic, algebra, geometry, problem solving or mathematical programming and they were available only in English or Spanish. In light of this, we see it as an important contribution to ongoing research into mathematics education to engage in projects that examine the process of developing applications for mobile technologies as well as studying the effects they could have on learning. Also, providing tools readily available for school teachers was an important factor for running the Apps in Math project, as detailed in the next section.

Design and implementation of the Apps in Math applications

The main goal of the Apps in Math project (AiM) was to develop 25 applications in 15 months for supporting teaching and learning mathematics in lower and upper secondary schools in Slovakia and Norway. In Norway, pupils have relatively good access to technology, compared with European countries. Almost 90 % of pupils use Internet in schools but the most common use is probably the computer and not mobile platforms. After school hours, as much as 94 % of all children aged 9-16 have access to a mobile phone, and 83 % have a smartphone. (Medietilsynet & Trygg bruk, 2014) Several schools have a policy of buying one laptop for each child in school. Most publishing houses have their own apps and games connecting to their textbooks, and there are usually many choices teachers can do regarding software for their pupils. Much is not translated into Norwegian, but this is generally not seen as a big difficulty.

In Slovakia not all pupils have their own smartphone or tablet; the further east one goes, the less pupils have their own mobile device (Michálková, 2016). In the primary and secondary schools – the typicality is to have three computer rooms per school, in which Informatics is mainly taught, so there is rarely room for mathematics lessons in these specialized classrooms. Pupils usually do not have their own PC. During 2013-2015, thanks to national project supported by EU funding, 22 000 tablets were given to Slovak schools, which usually meant set of 30 tablets per school. Pupils in one school are sharing those tablets; teachers bring them for lesson, at the end of the lesson pupils have to return them, because they will be used in other classrooms. In Google Play or App Store there are very few mathematical apps in Slovak language that are intended to be used in mathematics classes at lower or upper secondary schools. So there is a need for applications, which teachers could use in math classes and for different levels of schooling.

The applications (modules) developed within Apps in Math project focus on various mathematical topics that are part of Slovak or Norwegian curriculum for pupils aged 9-19. The development of modules went in coherence with Design based research (Wang & Hannafin, 2005) and its iterative cycles. The mathematics teacher educators from Trondheim and Bratislava have cooperated with academics and bachelor students of applied informatics at the Comenius University in Bratislava. Slovak bachelor students in Applied Informatics have programmed the modules based on the specifications from mathematics teacher educators and master and PhD students, as part of their bachelor thesis in informatics. The modules were tested extensively within the local participating groups in Slovakia and in Norway, as well as with pupils in Slovak and Norwegian schools. Reflective analysis of problems and obstacles was done and changes were implemented after each testing. All modules are part of one framework application called Apps in Math and they are divided into five main categories: Numbers, Functions, Geometry, Chance and Logic. Apps in Math is available for Android and iOS[1] platform and in Slovak, Norwegian and English language. Ebner (2015) has divided applications into four categories: stand-alone learning apps, game-based learning apps, collaborative apps and learning analytics apps. Apps in Math has the characteristics of being game-based learning application. Diah, Ehsan and Ismail (2010) have introduced the framework for mobile educational games consisting of four important segments: Learning Theories, Mobile Learning Approach, Games Development Approach and Learning and Education Medium. Most of the modules in Apps in Math apply the constructivism as the learning theory and for the mobile learning approach the games use activity-based themes for informal and lifelong learning.

Case studies

This section describes two case studies (Study 1 and Study 2) that were conducted to evaluate the effectiveness of mobile learning with Apps in Math application in real-world settings, with lower and upper secondary school pupils. We have chosen the SAMR-model for a quick categorization of the modules, where digital technologies can be placed on a scale from just replacing already existing practicing to facilitate types of tasks that could not have been done without the digital tools (Hudson, 2014). Limited resources and limited time made it necessary to choose for evaluation those modules that were closest to being finished. The module *Lucky Hockey* is based on the classic learning game *Green Globs* (Dugdale, 1982), and several versions of this game has been implemented over the years. The pupils who play the game are going to shoot a hockey puck across an ice hockey arena in order to collect as many coins as possible. The coins are shattered around the play field, sometimes in a random manner, sometimes to provoke a particular shot. The pupil shoots by entering a function expression, using the touch screen controls to alter the parameters of the function (Figure 2). Only linear functions are implemented, in contrast to Green Globs were several types of functions could be experimented with. Using the SAMR-model we can say that Lucky Hockey acts as a direct tool substitute, but that the functional improvement allows for a more dynamic and dual view of the representations of a linear graph and the corresponding expression. Hence we can say this app is an augmentation of traditional instruction.

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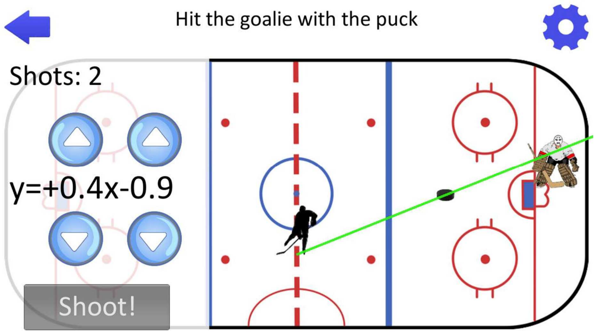
[1] http://www.project-aim.eu/eng/download



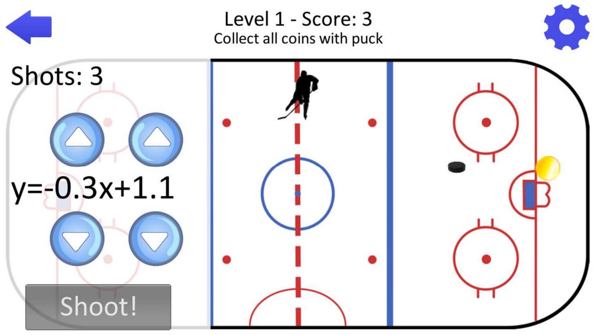
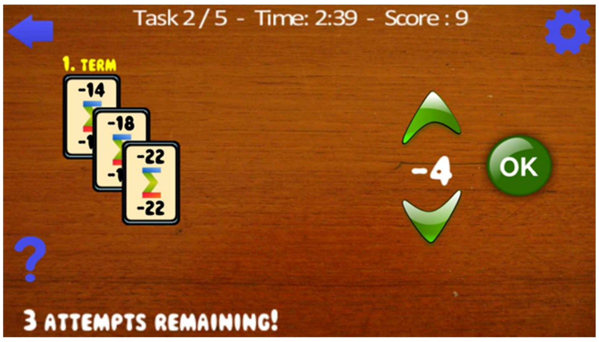


Figure 1: Learn mode of Lucky Hockey **Figure 2: First level of Lucky Hockey**

The module *House of cards* focus on arithmetic and geometric sequences in two separated submodules called *Arithmetricks* and *Geometricks*. By playing this game pupils should discover relations between the terms of the sequence and be able to write down basic formulae related to these relations. The number sequences are displayed on playing cards. Both submodules have a *Learn mode*, in which basic principles of the sequence are explained. The pupil has to determine the number, which is added/multiplied to/with each of the following sequence terms (Figure 3). The pupil has to answer five tasks correct within the time limit. After 3 incorrect attempts the correct answer is shown. In the next three levels the pupil should select the card, which belongs to the empty red spot in the given sequence within time limit (Figure 4). In the first level first 3 terms are given and the pupil should select the missing card for 4th and 5th term. Again, using the SAMR-model on Arithmetricks and Geometricks, we note that the effectiveness and readiness of the app makes work with sequences easier than in traditional teaching, or teaching done with real cards. Hence this app too provides an augmentation over traditional instruction.



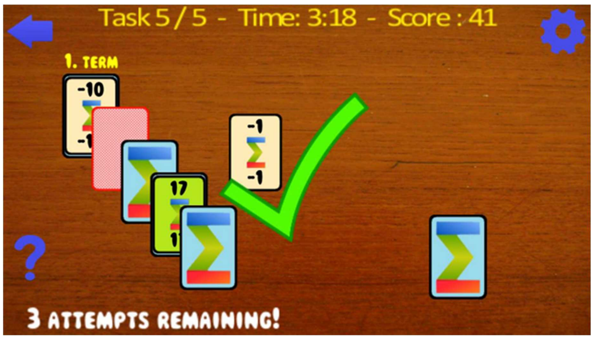


Figure 3: Learn mode of Arithmetricks **Figure 4: Third level of Arithmetricks**

The target group for the *Lucky Hockey* game study was Slovak pupils between the age 14 and 15 (grade 9), who had had no experience in linear functions yet. The goal of Study 1 was to determine which aspects of the linear function concept students seem to approach more effectively through the use of the Lucky Hockey game. Time limited gaming (25 minutes) was meant as an adidactical situation (Brousseau, 1997). The adidacticity was promoted by giving the students full responsibility for the technology-supported exploration of mathematical tasks by retroacting only with the milieu and not the teacher (Sollerval, de la Iglesia, 2015). All together 54 pupils from 2 different schools in Slovakia participated in Study 1 in November and December 2015.

The target group for the *House of Cards* game study was Slovak pupils between the age 16 and 17 (grade 11), who had not learned about sequences yet and had no previous knowledge about arithmetic and geometric sequences. The goal of Study 2 was to determine which aspects of the arithmetic/geometric sequence concept students seem to approach more effectively through the use of the House of Cards game. All together 49 pupils from schools in Bratislava participated in Study 2 in March 2016. They first played the Arithmetricks game (starting with Learn mode and consequently going through all three levels) for 25 minutes. The next lesson (in the same day) they played the Geometricks game with the same conditions. During both Studies 1 and 2 all pupils used an iPad. No pre-test was conducted since pupils did not have any knowledge on these topics. The post-tests were used to determine the level of acquired knowledge. All pupils of Study 1 and 2 completed the post-tests as part of the evaluation, right after playing the game. The phase of institutionalization took place a few months after Studies 1 and 2, due to prescribed curriculum.

A preliminary study was conducted in September 2015 with 77 pupils of different age (7 - 16), in order to introduce them the early versions of five different games, including the Lucky Hockey game. At this stage, the game was more or less fully working, apart from minor graphical issues. Part of the group (about 20 pupils) tested the Lucky Hockey game. During the testing pupils thought (while playing the Learn mode – Figure 1), that the expression is always $y = 0x + b$, because they were able to hit the goalie only by changing parameter b. This was an obstacle in Level 1, so we had to refine Learn mode and control the possible movements of a shooting player. Most of the pupils liked the game and did grasp the notion of linear function. In the preliminary study we also asked all the pupils about their interest in using smartphones or tablets to learn mathematics in school. Figure 5 shows their answers. 92.3 % of pupils, who answered positively on this question, also said that they would like to play tested games at home. Out of them 46.7 % in the situation when they are bored, 28.3 % for practicing mathematics and 25 % when doing homework.

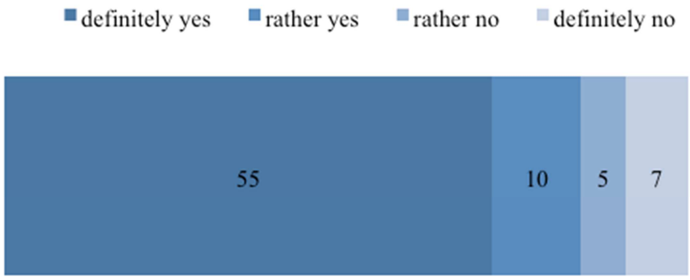


Figure 5: Interest of pupils to learn mathematics with mobile devices

Results

Figure 6 shows that pupils performed quite well in the post-test of Study 1. The average score was 5.11 and median score of 5. Pupils could obtain a maximum of 7 points, which were obtained by 13 pupils (24 %). Half of the pupils (50 %) scored 4-6.5 points, but there also was one pupil whose score was 0. The results indicate that most pupils understood the main principles of linear functions on an acceptable level. The lowest score performance they had occurred in the last task, in which they were asked to explain what impact the parameters a and b in the expression $y = ax + b$ had on the corresponding linear graph. Only 46 % of pupils explained it correctly. Nevertheless, they performed better in tasks in which they were supposed to draw a line in correspondence with a given equation (76 %) or select the correct line/equation out of four possibilities that is corresponding to a given equation/line (86 %).

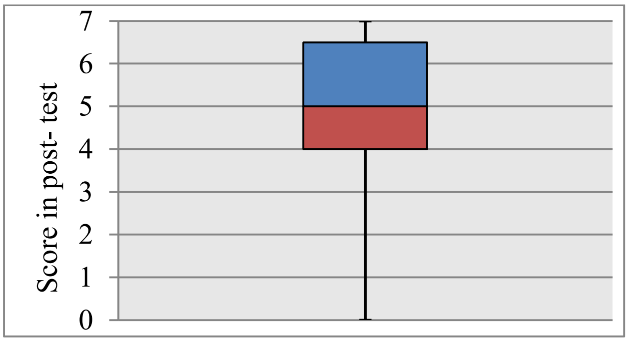
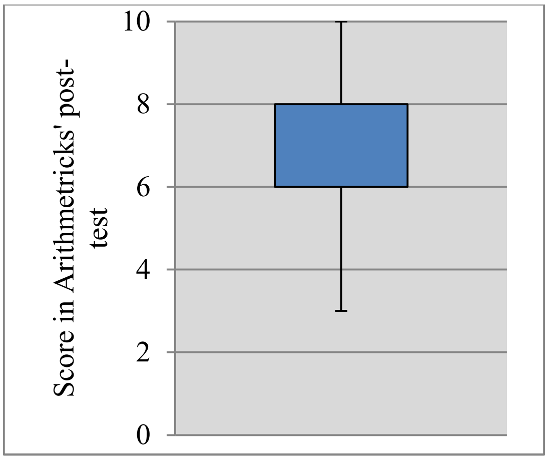


Figure 6: Box and Whisker Chart of Lucky Hockey Post-test Results

Figure 7 shows the results of pupils in the post-test of Study 2. It is clear that these pupils performed better in the Geometricks post-test. Here, 50 % of the pupils obtained 8-10 points, while 10 was the maximum. 18 pupils (36.7 %) obtained maximum score, and one pupil obtained the minimum score of 2. The second lowest score was 5, also obtained only by one pupil. The lowest performance was in the last task in which they had to write down the formula for how to find the 10th term, if they knew the quotient $\left(q = \frac{1}{3}\right)$ and the 1st term was given as a_1. Only 51 % of the pupils wrote the correct formula and explained their answer correctly. The most frequent error was made by 7 pupils (8.2 %), claiming that $a_{10} = a_1 . \frac{1}{3^{10}}$. In all the other tasks pupils were able to determine the unknown term, if they knew specified values of the 1st term and the quotient, or specified values of two consecutive or two nonconsecutive terms, with successfulness of 89 % - 100 %. The scores in the Arithmetricks post-test were slightly lower, with an average 6.82 and 6 as a median score. 8 pupils (16.3 %) obtained the maximum score and two pupils obtained the minimum score of 3. Distribution of scores within the box chart shows that approximately one quarter of the pupils obtained the same score, 6 points.



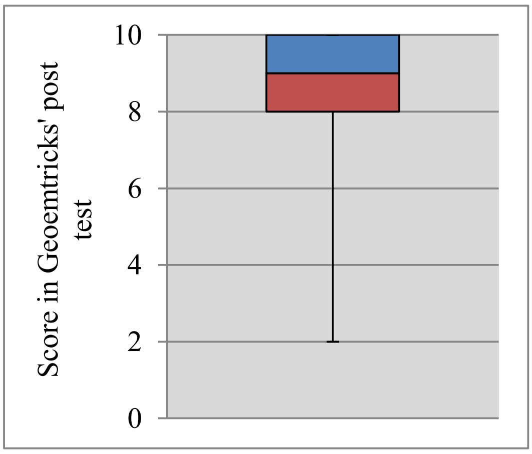


Figure 7: Box and Whisker Charts of Arithmetricks and Geometricks Post-tests Results

Discussion and conclusive remarks

We note from the results that most of the pupils did learn the main principles of linear functions or sequences at an acceptable level. But only about 50 % of the pupils were able to answer the last question correctly, which could be caused by not repeated and time limited playing of the game. This hypothesis also arises from differences between how the pupils scored in Arithmetricks and Geometricks post-tests. The pupils did play the Geometricks game after playing the Arithmetricks game and since the principle is not very different, it could cause that they performed better in the Geometricks post-test. As mentioned above, the phase of institutionalization took place 6 months after the Studies 1-2. While pupils who participated in Study 1 did not remember much about the linear function, it was different with pupils of Study 2. Pupils recalled the main principles of arithmetic/geometric sequence and told the teacher that it was not needed to explain it again: "It's like in that game we have played." From observation of the teacher we also note that traditional teaching of sequences went this time easier, probably also due to the mobile learning. The interest of pupils to learn mathematics with a mobile device was visible during testing both in Slovakia and in Norway. According to the results of the questionnaire it seems that most of the Slovak pupils would like to include mobile learning in their schooling. Testing of the other various applications from the project, not mentioned in this paper, also confirms that Slovak pupils and teachers consider mobile learning as a motivational way of learning and teaching mathematics (Michálková, 2016; Kapitulčinová, 2016). Mobile phone games in classroom is a novel idea and it might still cause the engagement of being a contemporary, "fresh" way of learning mathematics, which could be the reason of pupils' and teachers' enthusiasm. Hence, exploring the actual use of such tools are of high importance in mathematics education.

The results of Study 1 and 2 suggest that mobile learning can be both motivational for pupils when learning mathematics, and helpful when acquiring new knowledge effectively. We are still in the beginning of exploring the use of mathematical applications. We should keep developing appropriate didactical situations with applications that facilitate effective mobile learning. One lesson learned from this project is the difficulty of communicating mathematical ideas from the idea stage to the actual implementation. This became quite apparent when collaborating with different countries, different levels of study and different study branches. In the early days of programming for schools a Norwegian teacher named this the "kitchen-table-programming-era", where teachers did both the ideas and the programming. This produced ideas for exploring mathematics, ideas still valid today. In our experience this also explains partly why available mobile games remind us of the drill & kill games that has permeated schools for decades and continue to do so. Also, it turned out to be much easier to develop ideas with a narrow mathematical theme, than to make applications that facilitates exploration.

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