Abstract

It has been shown that children who control strategies are better able to direct their own learning and knowledge. Seeking for an effective teaching method to achieve this goal, we experimented with the Explicit Teaching method vs. a traditional school one, using both to teach the Working Backwards Strategy. The study was conducted amongst 57 mathematically talented students participating in a unique mathematics program called Kidumatica. A mixed method analysis showed that Explicit Teaching produced better results regarding students' ability to use the strategy, though it did not affect the students' ability to recognize the strategy. This indicates that young students can understand when to use this powerful tool and, with further guidance, can also master their ability to use it.

Keywords: Explicit teaching, Strategy, Working backwards Strategy

Introduction

Since Polya (1957), who claimed that students who control many strategies will become more effective and intelligent problem solvers, other researchers have advocated integrating problem solving strategies into school mathematics (English, 1993; Steiner, 2007), especially for talented students (Lee, 2014). The development and use of strategies is definitely not intuitive, and students need proper instruction, guidance and encouragement in order to systematically implement strategies in different domains - especially in problem solving (Tishmen, Perkins & Jay, 1996). Unfortunately, teachers face immense difficulties when it comes to teaching strategies in an effective way (Zbiek & Larson, 2015). This fact led us to look for an effective method to teach (talented) students some basic problem solving strategies. Based on interviews and talks with “professional mathematicians,” we decided on four such strategies: Trial and Error, Proof by Contradiction, Working Backwards, and Recursion, which were taught to students using the “Explicit Teaching method.” In this paper we focus on just one example, showing how Explicit Teaching can be used to teach students the "Working Backwards Strategy” for solving non-routine mathematical problems.

Explicit Teaching Method

Explicit Teaching is a systematic methodology that is currently used primarily to teach reading and mathematics. This method is described as “highly organized and structured, teacher-directed, and task-oriented” (Archer & Hughes, 2011). All stages of the learning process include mediation between the teacher and the learner, in which the teacher transmits an external understanding of
certain information to the learner, who then processes that pre-determined understanding (Olson, 2003). Nevertheless, using Explicit Teaching does not predetermine or confine learners’ thinking; on the contrary, it can help them become more active solvers and foster independent thinking (Portnov-Neeman & Amit, 2015). The methodology consists of a five step model (Figure 1). The steps described below are performed sequentially by the instructor in order to efficiently pass on specific information to the learner with as little ambiguity and room for error as possible (Rosenshine, 1986).

**Orientation:** Each lesson begins with a clear instruction about the purpose of the lesson. Learners need to understand what they are going to learn and how it connects to previous lessons.

**Presentation:** The lesson material is divided into small units that fit the learners’ cognitive abilities. The teacher uses a model or schema to guide them through their problem solving process.

**Structured Practice:** The instructor gives a direct and detailed explanation of the problem solving using the model or schema that was presented in the previous step. During this phase, it is critical that the instructor asks learners questions and encourages class discussion in order to check and assess their understanding of the material and clarify any confusion.

**Guided Practice:** In this practice the instructor addresses individuals’ questions and misconceptions one-on-one, and tailors responses to meet the individual needs of each learner. Students can work in small groups in order to develop their ideas together and help each other with the new material.

**Independent Practice:** In this step, learners are asked to complete an assignment on their own and without assistance. They are not expected to have a flawless understanding of the lesson, but they must understand the steps involved in the process. This step should continue until learners gain full independent proficiency with the materials.

**Figure 1: Model of Explicit Teaching**

‘**Working Backwards**’

‘Working Backwards’ is a useful and efficient strategy for solving problems in many aspects of our lives, in which an achievable outcome is known, but we have not yet determined the path towards achieving it (Newell & Simons, 1972; Portnov-Neeman & Amit, 2015). When dealing with word problems, for example, the information given in the problem can appear like a complex list of facts,
so it is sometimes helpful to begin with the last detail given (Wright, 2010). The Working Backward strategy is illustrated in Figure 2 and explained step by step in detail below:

1) Read the problem from beginning to end and identify all its components and steps.
2) Check the final outcome of the problem.
3) From the final outcome, start reversing each mathematical operation in each step until you reach the beginning of the problem. For example, reverse the adding operation and replace with a subtraction operation.
4) After reversing every step, resolve the initial state of the problem.
5) Check the answer by starting from the initial state and working through the steps to see if the final outcome is achieved (Amit, Heifets & Samovol, 2007).

![Figure 2: Model of the Working Backwards strategy (Amit, Heifets & Samovol, 2007).](image)

**Methodology**

The study presented here examined the effect of using the Explicit Teaching method to learn a new strategy, specifically the Working Backwards Strategy for mathematical problem solving. The research questions are: To what extent does Explicit Teaching affect:

a) The ability to recognize when the Working Backwards Strategy is needed for problem solving?

b) The ability to use and implement the Working Backwards Strategy?

**Context**

The study was conducted in the framework of "Kidumatica". Kidumatica - the math club for excellence and creativity - is an after school program for talented students in the 5th to 11th grades who are interested in mathematics, but require further tools to reach their full potential (Amit, 2009). Fifty-seven (N= 57) 6th grade students were divided in two groups: an Experimental Group (EG = 30 students) and a control group (CG = 27 students). Over a period of six months, these students learned different mathematical strategies, including the Working Backwards Strategy. The EG learned through the Explicit Teaching method, while the CG was taught using the traditional
school one. None of the students in this study had been research subjects in previous studies involving the Working Backward Strategy, and none had learned the strategy before.

**The ‘Explicit Teaching’ Group (Experimental Group)**

Students in this group studied all the strategies by means of the Explicit Teaching method. Each strategy, including the Working Backwards Strategy, was taught for four weeks by one of the researchers, according to the model illustrated in Figure 1. The teacher had an integral part in the lessons. She clearly and explicitly outlined what the learning goals are for the student, and offered clear, unambiguous explanations of the skills, information and the problem solving process. As the lessons progressed, the teacher’s role reduced, until students were able to solve problems independently. It was like riding a bicycle, were the instructor gradually releases his hold of the bike and the child rides off by herself. The first lesson started with an explanation of the strategy, including its importance as well as where and how it should be implemented. The teacher showed the students the model of the strategy and explained the role of each step in the solution process. The following lessons were dedicated to structural, guided and independent practices. During the structured practice the teacher gave a direct and detailed explanation of the problem solution using the Working Backwards model (Figure 2). The teacher encouraged discourse between the students and asked questions to assess their understanding and clarify any confusion. In the guided practice, students worked in smaller groups or by themselves on different working backwards problems. The teacher walked around the students and addressed individuals’ questions. When the teacher felt confident enough of a student’s abilities, that student was allowed to start working individually and begin the independent practice step. At the independent practice stage, students were asked to complete several assignments using the working Backwards Strategy, and to solve complex problems on their own.

**Traditional Teaching Control Group**

The control group studied the Working Backwards Strategy for the same period of time as the EG, but they studied the strategy in the traditional school method. This group differed from the EG in the following ways:

1. Lessons were mainly dedicated to students’ work. The teacher’s part was smaller than in the EG. Her role was to give short explanations about the lesson activities. She did not use the word “strategy” in her explanations, or explain that a special approach is needed for solving working backwards problems. Most of the lesson was dedicated to independent time, so that students would develop their own strategy toward those problems. It was important that students draw their own conclusions, create their own conceptual structures, and assimilate the information in the way that makes the most sense to them.

2. The teacher did not show the model of the strategy and did not name the strategy explicitly. Instead, students could develop their own model and meaningful name based on the teacher’s examples and their own experience.

3. The practice process in the CG was mainly independent, in contrast to the three levels of practice used in the EG. That led to less room for discussion and collaborative work between the students, unlike the EG, where time was allotted for these during the structured and guided practice.
Data collection and analysis

Data was collected via pre- and post-tests, students’ products, short interviews during and after the lessons, and teacher’s notes. The pre/post-tests were administrated to both groups before and after the learning program. Both tests included working backwards problems. This paper will discuss two problems from the pretest (problems 1 and 2 below), and two from the posttest (problems 3 and 4).

1. Card Problem: “Yael, Danny and Michael played cards. At the beginning of the game each one had a different amount of cards. Yael gave Danny 12 cards. Danny gave Michael 10 cards and Michael passed Yael 4 cards. At the end each one of them had 20 cards. How many cards did Yael, Danny and Michael have in the beginning?”

2. Mangoes Problem: “One night the King couldn't sleep, so he went down into the royal kitchen, where he found a bowl full of mangoes. Being hungry, he took 1/6 of the mangoes. Later that night, the Queen was hungry and couldn't sleep. She too found the mangoes and took 1/5 of what the King had left. Still later, the first Prince awoke, went to the kitchen, and ate 1/4 of the remaining mangoes. Even later, his brother, the second Prince, ate 1/3 of what was then left. Finally, the third Prince ate 1/2 of what was left, leaving only three mangoes for the servants. How many mangoes were originally in the bowl?”

3. Weight Problem: “Four students in the class weighed themselves. Cobi was 15 kilograms lighter than Adi. Gaby was twice as heavy as Cobi and Jenya was seven kilograms heavier than Gaby. If Jenya weighed 71 kilograms what was Adi’s weight?”

4. Baseball Problem: “The Wolverines baseball team opened a new box of baseballs for today’s game. They sent 1/3 of their baseballs to be rubbed with special mud to take the gloss off. They gave 15 baseballs to their star outfielder to autograph. The batboy took 20 baseballs for batting practice. They had only 15 baseballs left. How many baseballs were in the box at the start?”

At the end of each test, students were asked to write what method they had used to solve the problems. The purpose of the pre-test was to determine the homogeneity of the two groups. The post-test examined the effect of the teaching methods at the end of the learning process. A five point scale was used to rank students’ answers (5 points = full and correct answer, 0 points = no answer). For example, if students identified all the steps, calculated each one by doing the opposite mathematical calculation and wrote the final answer correctly, they received 5 points. Figure 3, for example, shows a five point solution for the “Weight Problem.” The problem has three steps: (1) Jenya was seven kilograms heavier than Gaby; (2) Gaby was twice as heavy as Cobi; (3) Cobi was 15 kilograms lighter than Adi. The student calculated the weight of each person by working backward through every step of the problem. Figure 4 shows an example of a 2 point solution. The student started with the last detail given and calculated Gaby’s weight correctly. However in the next two steps he did not reverse the mathematical operations and got an incorrect answer.
Findings

Findings from the pre-test showed that in both problems, there was no significant difference between the groups, which indicates that both groups had the same level of homogeneity. After six months of learning strategies, the average scores in the post-test for both problems were higher among the EG than the CG. In Table 1 we can see a significant difference in the post-test between the two groups in both problems. Figure 5 indicates that students’ ability to recognize the strategy improved after the learning process, but that both groups had similar results in the pre and post-test.

**Table 1: Results from pre- and post-test in the EG and the CG**

<table>
<thead>
<tr>
<th>Test</th>
<th>Problem</th>
<th>Group</th>
<th>N</th>
<th>Mean</th>
<th>SD</th>
<th>Sig</th>
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<tr>
<td>Pre test</td>
<td>Card Problem</td>
<td>EG</td>
<td>30</td>
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<td>Mango Problem</td>
<td>CG</td>
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<td>3.30</td>
<td>2.035</td>
<td></td>
</tr>
<tr>
<td>Post-test</td>
<td>Weight Problem</td>
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<td>1.977</td>
<td>$P = 0.96$</td>
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<td>Baseball Problem</td>
<td>CG</td>
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<td>1.74</td>
<td>2.141</td>
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<tr>
<td></td>
<td></td>
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<td>.183</td>
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<tr>
<td></td>
<td></td>
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<tr>
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<td></td>
<td>CG</td>
<td>27</td>
<td>2.89</td>
<td>1.553</td>
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</table>
Figure 5: Amount of students from the EG and CG that recognized the Working Backwards Strategy in pre- and post-tests

Our qualitative analysis of students' solutions revealed that the EG students reversed the mathematical operations much better and more easily than those in the CG, and were thus able to solve the problem correctly. Moreover, while the EG explicitly stated the name of the strategy they had used when asked, the CG students were very creative in naming the strategy, coining names such as, “going in through the back door”, “reverse manual” etc. Finally, the EG students used the model of Working Backwards Strategy in a very efficient way, sometimes adjusting the model to make it easier to use.

Discussion

Strategies are undoubtedly an important tool for goal-directed procedures in problem solving. Introducing them at a younger age can improve learners’ mathematical ability (Polya, 1957) as well as their understanding and thinking skills (English, 1993). To achieve this goal, it is important to use a specific teaching method (Tishmen, Perkins & Jay, 1996). In this study, that method is the Explicit Teaching method, through which we introduced the Working Backwards Strategy. The study examined the effect of this method on students' ability to recognize and solve working backwards problems. Fifty-seven sixth graders were divided into two groups, an experiment group (EG) that studied with the Explicit Teaching method and a control group (CG) that studied with a traditional school one. The strategy was unfamiliar to both groups and the findings from the pre-test showed that both groups had a similar starting point. At the end of the learning process, the group that studied explicitly showed higher results than the control group. The structured steps of the Explicit Teaching helped the students to have a better, clearer understanding of the strategy (Anhalt & Cortez, 2015). Qualitative analysis revealed that students who studied explicitly were more resourceful in their solutions. They understood how the strategy works, adopted it and changed it to make it easier to solve. We believe that this ability developed due to the discourse and the collaborative work in the structured and guided practice. We saw how students’ understanding of the strategy and its use improved over time. They asked more questions, listened to other students’ answers and learned how to avoid misconceptions. In addition, the integral role of the teacher in this method helped students gradually to build their confidence. Thus, these students were more prepared to work on working backwards problems by themselves. Our previous study showed that teaching explicitly can help students become active learners and foster their independent thinking (Portnov-Neeman & Amit, 2015). The current study supports this conclusion, showing that Explicit Teaching did not limit students' thinking by fixing it on a particular process. On the contrary, students understood the core principle of the Working Backwards Strategy and then applied it creatively in whatever way seemed best to them. Both groups showed improvement over time in their ability to recognize when and why the Working Backward Strategy is needed. The percentage of students that recognized the strategy before and after the learning process was similar. This is very encouraging, since it may indicate that the teaching method does not affect students' ability to recognize strategies. We can assume that with additional practice, all of the students could potentially master strategies and develop their understanding and their strategic approach to problem solving, but this has to be tested and researched. In this study, we experimented in ‘laboratory conditions’ with talented students, and found that the method works. Further research is
needed to confirm its effectiveness outside of the Kidumatica Mathematics Club, in the ‘real world’ of education.

**Conclusion**

Mathematical strategies are complex concepts to learn and understand, and we as educators must search for the most effective way to teach them. In this study, we used a systematic and structured methodology called Explicit Teaching, and found that students who studied with this method had higher scores than students who studied with a traditional school method. Introducing strategies like these to students is important, since they can help students evolve into better thinkers and develop their ability to solve problems. We believe that strategies can and should be introduced from a younger age so they can be developed over time. We have found that younger children are capable of acquiring the basic tools. Given time, they will be able to develop their tool kit of strategies further, and eventually master them all. It is our obligation as educators to teach our students how to use strategies correctly, and the sooner we do so the better.

**References**


