Exploratory study of teacher’s interventions in a modelling task

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The aim of this work is to analyse teacher’s interventions during the resolution of a modelling task. The modelling task used in the experience is intended to help students to explore concepts and procedures related to two-variable functions. During the resolution of the task the students will use iPads and specific software both to take and analyse data. In order to analyse teacher’s interventions we have designed an instrument that leads us to identify their influence in the appearance of learning opportunities.

Keywords: modelling, teacher’s interventions, learning opportunities

Introduction and objectives

Our research is situated in the modelling perspective on teaching and learning mathematics, in line with Lesh and Doerr (2003). In this particular framework the ideas of eliciting student thinking and help students to develop their emerging models are central to improve mathematical learning. That is why, in this work, we will focus in which teacher practices in modelling classes can foster student’s learning opportunities.

The difficulties related with the introduction of modelling in the classroom have been studied in several works (Blum & Niss, 1991, Schwarz & Kaiser, 2007, Borromeo-Ferri & Blum, 2013) promoting projects such as LEMA or MASCIL. Nevertheless, until now, relatively few studies have focused on teacher’s role during this process. Doerr and Arleback (2015) provide evidence of teaching practices that foster students’ independence as learners and modellers. In the same line, Stender and Kaiser (2015) investigate which kinds of scaffolding and intervention activities are adequate to promote independent student’s modelling activities.

The present work is an exploratory study that focuses in a particular kind of modelling task that consists on modelling a physical phenomenon using data collection with digital devices. This task has been already described in Diago et al. (2016). Students have to answer a specific question: How does intensity of sound distribute throughout our classroom? This initial question leads students to take some decisions, first related to the process of data collection, then to data representation and, finally, to data analysis. During this process the teacher, that was also one of the researchers, will try to act as a guide that promotes learning opportunities. So, our research question, stated in general terms, is: What teacher’s interventions can foster learning opportunities during a modelling task? More specifically, in this exploratory work our aim is a first identification of key aspects of teacher’s interventions that could foster learning opportunities during a modelling task. Our work is in line with the works by Doerr and Arleback (2015) and Stender and Kaiser (2015), but, in our study we focus in a particular open-ended task and in the role of the teacher during those moments that can lead to learning opportunities.

Our work has been carried out in different phases, first we have adapted the framework of learning opportunities developed by Morera (2013) in her PhD thesis (see also Morera, Planas and Fortuny, 2013) in order to fit it in the context of modelling (because Morera’s works refers to geometric tasks that are not necessarily open, nor in the resolution process, neither in the solution). Then we have designed the teaching experiment, in particular we have focused in the teaching sequence and
in the teacher’s role. The last part corresponds to the analysis of the experience in order to obtain the results of the exploratory research.

**Theoretical framework**

**Learning opportunities**

Following Morera (2013), we will consider that learning opportunities appear during the resolution process of a task. In particular, they appear when students have to reorganize their conceptual structures, that is, increase their connections or relate them with the learning of new procedures. There are some “good practices” –such as the use of digital or manipulative tools and the implication of students in a research– that can improve the development of mathematical learning opportunities (Vermont report, 2000). Moreover, collaborative work promotes learning opportunities. That is why we observe which teacher’s interventions can promote these opportunities when students when they discuss during the resolution of a modelling task using digital devices. For this aim, following Morera (2013), we will categorize learning opportunities:

1- **Conceptual opportunities**: correspond to those situations that foster the construction of a self-concept.

2- **Procedural opportunities**: correspond to those situations that strengthen the learning of procedures. This implies that students know how to do but also when and why they do.

3- **Management opportunities**: correspond to those situations that promote the need of some skills (mathematical, social, etc.) in order to implement a task.

This categorization will be used in our analysis in order to relate them to teacher’s interventions during the resolution process of a modelling task.

**Modelling**

Mathematical modelling, as a teaching approach, has been described in several perspectives and contexts in the literature. Our analysis is based on the modelling cycle of Blum & Leiß (2005), these authors describe modelling process as a chain of activities, starting from a real situation. There are different perspectives to describe the modelling process. A commonality between all perspectives is the existence of a real situation problem, which has to be simplified by the problem solver. The reality has to be mathematised, that implies its data, concepts, relations, conditions and assumptions have to be translated into mathematics (Blum & Niss 1991).

In the work presented by Doerr (1997) the author develops a theoretical framework for an approach to a modelling experience. In this approach we identify important aspects that have been considered in our work: the starting point is a physical phenomenon that will be identified through a simulation in order to be analysed using technological tools. These aspects have been the key to design our modelling proposal.

As stated by Geiger (2011), the use of technological tools enriches the modelling process, as mathematical routines and processes, students, and technology are engaged in partnership during the solving of the problem. Starting from a real problem the students will be able to elaborate both a personal representation and a personal concept of two-variable function, which will contain precursor elements of the mathematical concept. In particular, we are interested in the Cartesian 3-dimensional representation of two-variable functions. In the experiment, the pupils will make a mathematical approach to concepts related with sound physics and obtain a mathematical model of the sound intensity distribution through the classroom from a punctual sound source.

Blum & Borromeo (2009, p. 54) state that there are not master rules to introduce modelling in classroom, but, following their experience they are able to recommend some effective practices for teachers:
a) It is important to maintain a “permanent balance between teacher’s guidance and students’ independence”

b) “It is important to support students’ individual modelling routes and to encourage multiple solutions.”

c) Teachers need specific formation about management of a modelling task in order to be able to use a broad spectrum of intervention modes or strategic interventions.

d) Teachers should know ways to support adequate student strategies during the resolution of modelling tasks.

Since the teacher that has conducted the exploratory experience is also one of the researchers, she has kept in mind those recommendations.

**Methodology**

In this section we will develop the methodology used during the experience and the analysis. In the first part we will explain the context of the exploratory study and the design of the teaching sequence, and in the second part we will explain the analysis procedure of teacher’s interventions.

**Teaching sequence design**

The teaching sequence is integrated as an extracurricular project in the school calendar. In order to design the teaching sequence, we have considered an authentic context (the classroom) and a real problem (where is the best place in the classroom to hear the teacher?). In order to develop the teaching sequence, we have adapted the sequences of activities described by Schoenfeld (2013) as follows:

a) Task introduction: the teacher poses a research question to the students (how sound intensity is distributed in the classroom) and organizes the class structure (the students form 4-people groups)

b) Exposition: during the first class the teacher defines the problem and introduces some basic aspect about sound’s physics. In the beginning of the rest of the sessions the teacher just explains the objective of the day and, finally, she briefly summarizes the concepts that have been worked.

c) Whole group discussions: in order to manage the debate between the whole student’s group, the teacher asks some guiding questions.

d) Small group work: students work and discuss in small groups (intra-group debate).

e) Student presentation: students expose their work to their fellows.

f) Post-lesson analysis: during this phase, the teacher highlights the most important mistakes in order to avoid them in the future, moreover, the whole group will share those ideas that lead to a solution.

The implementation of the modelling task was done in a group of 32 tenth grade students in Valencia, Spain. We chose this group of students because there had no particular experience with modelling, moreover no mathematical concepts related to two-variable functions are present on the syllabi of the corresponding level for these students. During five sessions, the students act as researchers, organized in five groups of six or seven students each. They have to answer a specific research question: *How does intensity of sound distribute throughout our classroom?* Pupils use the iPad® as the main digital interactive tool both for the qualitative and quantitative analysis of the phenomena. In what follows we describe each of the session in detail. The distribution of the sessions has been designed following the objectives of the exploratory research. We will try to foster learning opportunities related to different phases of the resolution process in each of the sessions of the experience.
The first session, the most theoretical of the experience, was devoted to the introduction of concepts related to sound and instrumental tools. Pupils use the iPad® as sound meter in the research process. We chose the app Decibel Ultra Pro® ([http://www.dev-apps.de/app-decibelultra.html](http://www.dev-apps.de/app-decibelultra.html)) in order to measure real data from a sound source. During this session some instructions about the app were explained to the students.

During the second session the students discuss, in small group work, about the discretization of the classroom in order to obtain a sound map of it. In groups, students think about the optimal sound intensity measurements sample covering all the classroom space. The idea is to depict a sound intensity map of the classroom from the punctual sound source. The sound arises from a computer located on the teacher’s table. During the last part of the session the teacher conducts a debate to lead students to choose the optimal way to discretize the classroom space in order to take the measures in the third session. Figure 1 shows the selected grid pattern of the class after the group discussion. It consists in a discretization of the classroom in cells where the sound intensity will be measured during the third session. The image on the left corresponds to the classroom plan and on the right one we have added the grid; each point of the grid corresponds to the exact location where students will measure the sound intensity.

![Fig. 1 Selected measurement points of the classroom](image)

The third session is devoted to the data gathering, making sound intensity measurements for each cell of the selected division of the classroom. In advance, the teacher prepared the classroom in order to empty all the tables and chairs and mark on the floor the points where measure has to be taken. In this session it is relevant to introduce some ideas related to measure of physical magnitude that reveal the importance of using arithmetic mean in the process. The teacher, before the measurement process, fosters a debate with the whole group in order to introduce the idea of the measurement errors. The students, in order to minimize measurement errors, decided to take four measures in each point and use the arithmetic mean of these values to have a more accurate value of the sound intensity.

The fourth session is devoted to the representation of the sound intensity data. The data is collected in table format. During this session the students mainly work by groups. Teacher provide pupils with a physical structured material, Multilink® cubes, to give them the opportunity to ideate a three dimensional representation. It is important to remark that we don’t aim in this teaching unit the elaboration by pupils of the Cartesian representation of three dimensional space as an ordered triple \((x, y, z)\). Nevertheless, our aim is to give students the opportunity to elaborate a representation which could be an intermediate step towards the Cartesian representation. Such representation is a three-dimensional extension of the concept of *applicate* used by of Euler (1797, t. II, p.5) in his work about two dimensional representations of a real function. Students are acquainted with a...
representation in some sense similar to Euler's applicates: bar graphs. At the end of the session the teachers highlight some of the results obtained by students and lead them to interpret the representation.

The last session was devoted to the elaboration of two dimensional representations of data. For this aim the students used the online collaborative tool Plotly® (https://plot.ly/) working by groups. This tool allows students to obtain heat-maps and contour-maps easily. At the end of this session the teacher summarized all the performances and concepts elaborated by the students and reveal the importance of the mathematical modelling in the understanding of the real-life phenomena.

Fig. 2 Three dimensional and two dimensional representations of the data

Analysis’ tool

During the experience, each group of students had an iPad in the table to record their discussions and the teacher had also a sound recorder during all the experience. All the sessions have been also video-taped with two cameras situated on opposite corners of the classroom. In order to analyse the teacher’s activity during the development of the task we have transcribed the discussions between students working in small groups and the teacher.

Then, in the first part of the analysis, we have identified the different aspects of each teacher intervention. First, we have identified the objective of the intervention (clarify, motivate, explain, recapitulate, etc) following the definition of different kinds of objectives used by Gallart (2016) and Gallart et al. (2015). But, in fact, we have observed that the effectiveness of a teacher in the classroom depends not only on the objectives of their interventions or even on his knowledge. In fact, it is also important to consider other aspects such as the ability to combine formal and informal language, authority or permissiveness during the discussions or even the management of the time. That is why, for teacher’s interventions, we have also analysed the tone (low or high), the language (formal or informal), the conviction (determined or dubious), the clarity (explicit or implicit) and the role of the teacher during the intervention (expert, moderator, observer, adviser or resource’s manager).

Since teacher’s interventions occur during the resolution of a modelling task, we have also considered, in order to better analyse the group discussions, in which phase of the modelling cycle (Blum and Leiß, 2005) were working the students.

Moreover, for each of the five work sessions, we have identified the different learning opportunities that have appeared, distinguishing between procedural, conceptual and management opportunities.

In order to better explain our analysis tool, we will describe the analysis that has been made for one of the group discussions. First, we will detail the transcription of the discussion. This fragment corresponds to the second session, students debate about how to organize the measurement, they decide in which points of the classroom they have to measure the sound intensity. We have translated the transcription; the Spanish version can be found in Donat (2016).

Student A: We must divide in parts. How many?

Student B: We divide in three parts. I would put in the corners and behind.
Student C: To divide in zones is fine... but I don’t know exactly which zones...

Student A: But, you will leave the chairs and... [Teacher arrives]

Teacher: The problem is not whether there is someone or nobody sits there. We have to see how the sound is distributed in the classroom independently of people and, the most important, to see the variations

Student B: Maybe if we measure in front, in the centre and behind, we will see if it varies a lot...

Student D: Yes, this can be a good idea, but, just one?

Student B: I don’t know, three or four points

Student A: And... why?

Student B: I don’t know... Because the classroom is big!

Student A: OK, put it there and we explain it because I think it is fine. With these measurements we can observe differences of sound intensity and that is the question.

In order to better analyse the transcription, we have designed a diagram to resume in which phase of modelling cycle the students are and how the intervention of the teacher affects the debate. Squares corresponds to students’ interventions and the curve arrow corresponds to the teacher intervention, the numbers in the squares correspond to the numbers in the modelling cycle by Blum and Leiß (2005): 1. Understanding, 2. Simplifying/Structuring, 3. Mathematising, 4. Working mathematically, 5. Interpreting, and 6. Validating.

![Diagram of a particular episode](image)

In this episode, the teacher speaks with a high tone and the language is formal. We can distinguish an implicit part (at the beginning of the intervention) and an explicit one. We guess that the teacher acts as an expert who wants to get students though back into lines and also to emphasise the objective.

The vertical strips in the squares of the diagram mean that the mathematical strategy of the students is not correct from a mathematical point of view; diagonal ones mean that there are some improvements in the mathematical reasoning. In the complete analysis we have also used horizontal strips when the mathematical argument is correct.
Exploratory results and conclusions

In this experience we have analysed whole group and little group discussions, the complete analysis can be found in Donat (2015). A first look to the different diagrams obtained from the analysis of the transcriptions give us a positive view about the teacher’s interventions during the exploratory experience. In fact, in almost every case, we observe an improvement of the quality of mathematical arguments used by students or even a positive change of strategy. We deduce that correct teacher’s interventions can promote mathematical learning opportunities.

Those learning opportunities are significant when the teacher invite students to participate and gets them though back into line. These interventions are the most regular in our experience. We guess that this is in this way because, when the teacher appears during a debate in a group, students make an effort to explain clearly their strategies and, at the same time, they think about them in order be clear and easy to understand. When they discuss with the teacher, students can more easily detect if something does not fit in their reasoning. When teacher get students though back into line (acting as a resources’ manager), students interpret their results and, in some case, they modify their resolution plans or even they reject it. Other interventions that have contributed in some sense to students for having learning opportunities have been the comparison or the reflection about the development of the task.

There are also some evidences of differences between intra and extra group debates. In the first case, teacher tends to act as a resources’ management or even observer. Moreover, during the intra group debates we find that teacher interventions lead to foster procedural and management learning opportunities. However, in the whole group discussions (extra group debate), teacher tends to act as moderator and mainly foster conceptual learning opportunities. In most cases, the teacher prefers to give implicit information that can be suggestive to let students self-manage their strategies. When teacher speaks to students she tends to use different tones, alternating low tone with high to catch the attention of students.

This exploratory work confirms that the implementation of a modelling task is, also from the teacher activity point of view, a very complex task. We have detected several aspects about teacher interventions that can lead to deeper studies. In particular, we consider that is it very important, at least in Spanish context, to research about the teacher training in order to introduce modelling in the classrooms. This is, in our country, an open problem.

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