French engineers' training and their mathematical needs in the workplace: interlinking tools and reasoning

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This paper concerns the theme of "mathematics in the workplace" in the context of engineer work in France. In the continuity of recent research, it draws results on a two steps enquiry (questionnaire and interviews) on 237 French engineers. With the point of view of the Anthropological Theory of the Didactic (ATD), I study questions concerning the praxeological mathematical needs encountered by these engineers in their daily work ("in the workplace") and about their mathematical training and its adaptation to these needs depending on the training institution. It shows that math should not only be considered as a "tool" (that it is anyway), because engineers sometimes need to have an accurate understanding of what they use. Furthermore, it shows that the Preparatory Cycle has a great impact on the future of the engineers' mathematical abilities.

Keywords: Engineer Schools, Mathematics in the workplace, Praxeologies, Preparatory Cycle.

Introduction - Context

The field of "Mathematics in or for the Workplace" has recently received an increasing interest especially at Tertiary Level (Biza, Hochmut, Kakhbaz, & Rassmussen, 2016). Worldwide researchers have contributed to think about and beyond dichotomies such as "school versus work" math (Bakker, 2014). In the case of engineering apprentices, Ridgway (2002, p. 189) shows that "mathematical challenges of engineering differ from the mathematical taught in school. In particular, great precision is required, applied to variety of mathematical techniques; a good deal of practical problem solving is necessary". Hochmuth, Biehler and Schreiber (2014) go further considering differences between mathematical practices in higher mathematic lectures and in advanced engineering lectures. They highlight the idea (p. 697) that for "solving a specific task, (engineering) students have to make specific decisions regarding the relevance of knowledge". Kent and Noss (2002, p. 1) have identified "a pattern of mathematics-in-use in which mathematics of school (are) transformed in something rather different, […] part of a social practice", and Romo-Vázquez (2009, p. 37) adds that "their most advanced dimensions tend increasingly to be supported either by experts or by software" and that "the needs of non-specialists seem to move towards the ability to manipulate these mathematics as a tool for communication through specific languages" (p. 37). All these works evidence that the usual training received by future engineers is not always adequate, depends on the kind of training institution and that their mathematical needs are complex.

In the following, I investigate similar issues in the context of engineer education in France. In this country, "engineer schools" are independent institutions, not inserted within universities. For becoming an "engineering apprentice" in an engineer school, students first have to follow a two years "Preparatory Cycle" after the baccalaureate. Those two years studies can take place in different kinds of institutions:

- CPGE (Preparatory classes, Classe Préparatoire aux Grandes Écoles): This is a demanding training that concerns 50% of the future engineers. It takes place in "Lycées" (upper secondary schools) and has been historically created to allow students to enter the most prestigious engineers schools. The curriculum is rather generalist, and the admission very selective.
- CPI (Integrated preparatory cycle, Cycle Préparatoire Intégré): for nearly 25% of the future engineers, this training takes place directly in the engineer schools. The curriculum is more adapted to the specialty of the school (Mechanics, Chemistry and so on); the admission is also selective.
- University: the remaining 25% of the engineers follow their Preparatory Cycle in classical Universities (no selection for admission).

In this paper, after a presentation of my theoretical framework, situated within the Anthropological Theory of the Didactic (ATD), and of my research questions, I expose the methodology and the details of my enquiry. It comprises two elements: an online questionnaire submitted to working engineers, and semi-structured interviews with some of the respondents. I analyze the answers to selected questions of the questionnaire and then the interviews. Finally, I discuss these results and present some perspectives.

Theoretical framework and research questions

Mathematics practices in the workplace are conducted by the needs of the workplace itself. The diversity of existing tasks added to the particular tools and resources used in each workplace tend to make a research generalization difficult. Moreover, it is recognized that "school mathematics are often obscured by the production goal, technology, artifacts and established routines of workplace activity" (LaCroix, 2014, p.158). Furthermore, speaking of "school mathematics"requires making a difference again between the institutions where the training has taken place. For these reasons I have chosen an institutional perspective, provided by ATD (Chevallard, 2006).

I use in particular the praxeology concept that is a system $[T, \tau, \theta, \Theta]$ designed to model every human activity (i.e. a certain subject's activity in a certain institution). Among the four elements of this organization, we first find the tasks type T. The observed task type T is associated to a technique τ to create the "practical-technical block" (so-called a *know-how*). The second block is called "technological-theoretical block" formed by a technology θ (meaning a rational discourse that justifies the technique that is used) and a theory Θ, which role towards the technology is the same as the one of the technology towards the technique I am interested in "mathematical praxeologies", which means here praxeologies where mathematics intervenes in on or several components of the praxeology. In Chevallard's theory, praxeologies can moreover be adapted from a very general to a very precise point of view, following "codetermination levels" that I do not detail here. Using this approach, the two research questions I study in this paper are the following ones: (1) Which mathematical praxeologies live in the "workplace" institution for French engineers? (2) In which institution did they learn the mathematics they use in the workplace?

Methodology

The first step of my enquiry is an online anonymous questionnaire addressed to working engineers. I have sent it to institutional mailing lists (more than 20) of former French engineering students. To be as relevant as possible, I have tried to spread this questionnaire in schools with different domains of specialty such as data processing, electricity, electronic, agronomy, financial, chemistry, mechanics, materials etc. In fact, I was not able to know in advance the number of engineers that would receive the invitation to participate, nor how many of them would answer.

The questions mostly deal with the training the engineers have received in math and the questionnaire is divided in four parts. Only the first three ones will be analyzed in this work:

- The first one is about personal and professional elements.
- The second part is to precise what kind of praxeologies they have encountered during their training in mathematics in their engineer school.
- The third part is about their effective use of mathematical praxeologies: Is math a real need for their job? For what type of tasks do they need math most of the time? For what professional objectives? Have they had a continuous or self-training after their engineer school? What difference with the techniques of the initial training? What kind of tools: software, books, community, lectures notes, MOOC, etc.?

The second step of my enquiry has consisted of semi-directive interviews with 6 engineers selected according to their responses to the questionnaire and representing different classes according to the following variables: age, gender, institution of preparatory education and domain of specialty (see Figure 1).

	John	Peter	George	Matthew	William	Alice
Age/Gender	25/male	27/male	35/male	29/male	35/male	30/female
Qualification	Computer	Computer	Materials	Chemistry	Electricity	Materials chemist
Domain of work/job	Signal (audio) processing	Data security	Consultant	Control process engineer	Entrepreneur in financial analysis	Motorcars development engineering
Preparatory Cycle	CPGE	University	CPGE	CPGE/University	CPGE	CPGE

Figure 1: The six engineers interviewed

I describe here briefly the four parts of the interviews: The first one is about the opinion of the engineers on their own training (preparatory and engineer curricula) regarding their today specific mathematical needs: what seems to them well adapted or not and why? Based on the same idea, the second part is aimed to try to make them propose something that should be or should have been taught in their training, how and why. The third part is about their view about the autonomy concern; I do not use it in this paper. The fourth part is about their self-training for learning specific mathematical useful praxeologies: which devices or resources? What difference with their initial training?

Questionnaire's answers analysis

237 engineers from all over the country have filled this questionnaire, some of them now working abroad. In part 1, I observe that the predominant represented domains of activity are Chemistry, Physics Materials and Energetic, Computer, Electrical and Electronics, Production and Mechanics, Generalist, Agronomy and Economy. The repartition according to the principal variables is as follows (Figures 2, 3 and 4):

Age	Min	Med	Max	Avge
Years	24	29	61	32

Figure 2: Age repartition

Gender	Women	Men
%	38	62

Figure 3: Gender repartition

Preparatory cycle	CPGE	CPI	Univ
%	68	12	20

Figure 4: Preparatory Cycle repartition

In part 2, question 10 (have you received a training in mathematics in your engineer school?), 183 engineers amongst the 237 (77%) have answered yes. Among the 23% others, we can observe that 83% are chemistry engineers. This may show that the mathematical training depends on the precise orientation of the studies.

Question 12 (During your training in engineer school, the main mathematical contents taught were…) concerns the mathematical contents mostly taught in the engineer schools, I have proposed a list of main mathematical themes. I chose those themes according to groups of chapters mostly found in math literature for engineers, and the results are in Figure 5:

Contents	Scientific computation	Analysis	Algebra	Probability	Statistics	Modelling	Logic	Set Theory	Graphs
%	40.4	47.5	44.8	68.9	84.7	27.9	23.5	17.5	16;9

Figure 5: Mathematical contents taught in engineer schools

In Figure 5, I notice the score of Statistics and Probability: it seems to be the most common mathematical theme taught in the engineer schools in France, after Analysis,.

In part 3, question 19 (Would you say that you encounter (or have encountered) a real need of mathematics in your engineer's job?), 53% declare that they don't have a real need of math. In the next question (question 20), like in question 12, I have proposed a list of main mathematical themes used in the workplace the results are in Figure 6.

	Scientific computation	Analysis	Algebra	Probability	Statistics	Modelling	Logic	Sets Theory	Graphs
%	69.4	44.1	25.2	37.8	55.9	49.6	54	9	18

Figure 6: Main mathematical contents needed

In figure 6, the Scientific computation domain reaches the highest level. Then come Statistics but with a far lower result compared with Figure 5; same for the Probability, the Algebra and the Sets Theory domains. On the contrary, according to those percentages, the domains of scientific computation, Modeling and Logic seem to represent important needs although they are not taught widely. In the answers to question 21 (For what kind of professional tasks?), the engineers explain the practical use of these contents. The main tasks mentioned are simulation, modeling, data analysis, software or algorithms development, basic calculus for estimates, budgets, chemical dosing…

Interviews analysis

In this section I try to observe, drawing on sections 1 and 4 of the interview, the mathematical praxeologies present at the workplace, according to the interviewees. I recall that I consider as a "mathematical praxeology" a practice, and a discourse commenting/explaining this practice, where mathematics intervene. I propose a classification of these praxeologies, and I also try to identify in which institution the mathematics involved were met.

Transversal types of tasks and mathematical technologies

I classify in this category praxeologies of the workplace where the type of task is general, not necessarily linked with mathematics (as we will see below, it can range from “problem solving” to “communicating”); and the engineers mention mathematical techniques, and even more importantly technologies in the corresponding praxeology.

Some engineers identify, in the workplace, "reasoning" or "problem solving" type of task directly linked or not with mathematics (e.g. making an estimate of costs). Those coming from CPGE declare that, for such tasks, techniques and technologies they learned during this preparatory cycle are useful. The techniques and technologies they cite are linked with proof, testing hypotheses or logic. Obviously these techniques and technologies have been met in CPGE for very different types of tasks, but these engineers have transferred them to the workplace. For instance, John says that proof, seen as a method in CPGE, is very important to him in his job because it makes him understand the utility of mathematical rigor. George explains that, as a projects manager, he has to understand the mathematical thinking hidden behind a phenomenon more than the phenomenon itself. William says that the prominence of hypotheses verification in reasoning is what sometimes makes the difference between him and some of his colleagues, as well as being able to rigorously check the result of this reasoning at the end. Finally, Alice has told us the importance of logic in her everyday job. She gives the example of the contraposition: when she had been taught this kind of logical reasoning in CPGE, she thought it would be useless for her. Years later, when she had to work on "experience plans", she has realized that it is very important to master it when trying to show that an implication is true or false.

According to the declarations of the interviewees all of the mathematical contents corresponding to these daily needs are taught especially in CPGE more than any other institution.

Another kind of transversal mathematical praxeology is what John, Matthew and William are referring to as "basics", corresponding to 'basics skills' used by Ridgway (2002). The task types in the workplace are situated in many domains like cryptography (Peter), iterative problems solving in computing (John), actuarial science (William). Because of the variety of tasks, it is also difficult to identify comprehensively all the techniques (integrating, solving equations or ODE, etc.) and technologies (functions of several variables, geometry, matrices) in use. One important type of task appearing in the interviews can be formulated as: “Meeting and understanding new concepts”. For this type of task, having a good general knowledge in Analysis and Algebra, including theoretical aspects, is mentioned as very helpful. This can be seen as an evidence of the theoretical bloc of praxeologies in action.

In a similar way, I identified in the interviews the type of task: "communicating on mathematics". George declares that, thanks to his training in CPGE, he feels at ease to communicate on math subjects with the people he works with. In this case the type of task is directly related with mathematics, and the techniques for presenting mathematics have been learned in preparatory classes. Another type of task cited by George is "Exploring new domains" like, for instance in statistic physics "branch prediction". I observed the same type of task for Matthew and William in other domains like computation or finance. For this type of task their initial training in mathematics is not sufficient, and brings to "searching on the Internet"(forums, specialized websites). Sometimes they have a look into their old lectures notes or in books as mathematical references that they need anyway to be able to enter the field. For this way of learning, they say that they feel satisfied to find the right information by themselves.

Types of tasks in specific domains and mathematical techniques

In the interviews the six engineers also describe types of tasks met at their workplace but belonging to scientific domains, like physics; the techniques in the corresponding praxeologies include mathematics. In these praxeologies I did not clearly identify technologies. This is the second type of mathematical praxeologies I observe in my analysis.

First, I would like to highlight the fact that basic mathematical skills are also mentioned as providing techniques for many specific types of tasks in various domains, like for example the task "modeling the ageing performance of a material" (Alice). Nevertheless, the principle of use of the techniques and technologies differs: the aim is to be able to use some results (like theorems or formulas) without trying to understand them mathematically. Most of those basic skills are taught in the Preparatory Cycle, but the techniques (and technologies) they provide for the workplace are taught in the engineering schools. In fact these types of tasks are well known for many years; the same holds for the associated techniques.

Amongst these basic skills, the case of Statistics and Probability seems specific because this domain is mostly not taught in the various Preparatory Cycles in France. Each engineering school provides its own specific training adapted to its needs. According to the interviewees, once confronted in the real world of the workplace, sometimes a statistics formula becomes useful (they mostly remember having learnt at the engineer school a lot of theory which does not intervene in their work).

Reasoning+Using="Reasusing": a concept for a personal and new mathematical experience

A last category of mathematical praxeology I found out in the interviews combines mathematics in the techniques, in the technology and even in the theory. This seems to be linked to specific types of task, requiring the development of original techniques – almost a research work. John cites a type of task that can be formulated as: “outperforming competitors in the design of a new software”. He explains that he has to know which theorem he must use, but not only: he also has to have a deep understanding of the proof of this theorem to be able to understand which parameters will allow him to obtain a result in a smarter way than other colleagues. To illustrate this, he gives the example of audio latency that is one of the most important qualities for the client of music production software. The type of task here could be "Reduce the latency". It corresponds to a short period of delay between when the musician plays and when he can hear the sound through the sound system (e.g. headphones). When the competitors offer a 20 milliseconds latency, John has to put his efforts to find in the right theorems (mostly based on Fourier Analysis) how to minimize it to 6 ms. This will make the commercial difference and it requires that he really understands what is happening "inside" the theorem. This corresponds to the technique "analyze a theorem proof". I consider this as a third type of mathematical praxeologies with a type of task requiring some innovation.

Discussion - Conclusion

Drawing on the results exposed in this paper, I now come back to the two research questions presented above.

About the mathematical praxeologies that live in the “workplace” institution for French engineers, the primary result in this study is that only 47% declare they have a real need of math in their everyday job. Going further inside this fact, I have encountered three different kinds of praxeologies: A first one with a general type of task, like “solving a problem” or “communicating”; techniques, and mostly technologies involving mathematical elements like reasoning and proving, and also some elementary mathematical skills. Rigor, logic and an amount of math basics (sometimes considered as useless at first sight, because lacking of concrete sense to them) are necessary for the everyday work of the engineers, and also allow them to communicate more easily with other people in their working environment. The second kind of praxeology that lives in the workplace comprises specific tasks (simulating, modeling, data analyzing, calculating, etc.) associated with mathematical techniques: here again, the math basics are considered as very important but they are seen as providing techniques. The last and rather interesting kind of praxeology is the mix of reasoning and using (I call it "reasusing"): for an engineer, it means to articulating the technology or even the theory to make them become an integrated part of a technique for a specific kind of mathematical type of task (such as a logical analysis of a situation, understanding a theoretical mathematical concept).

For the second research question about the institution where they learn the mathematics they use in the workplace, I notice that the praxeologies developed in all types of Preparatory Cycles are mostly based on teaching of mathematical basic skills. To end this analysis, I must highlight that the engineers who declare needing the first kind of praxeologies (thinking, reasoning and problem solving) that where taught during their Preparatory Cycle are all coming from the CPGE institution.

Finally, my study certainly has some limitations. It cannot be considered as fully representative of the French engineers population (in terms of age, gender, domains of work, and Preparatory Cycles). Moreover a large part of it is based on what the participants *say* about the mathematics they have learned and use, but it is not clear that they all have in mind the exact same interpretation of things. I will work on this issue in my future research.

But the results that I expose can lead us to think that even if an important part of the engineers don't really need them daily, they don't consider mathematics exclusively as providing techniques. Receiving a training type "math as a toolbox" is not satisfactory for them because they sometimes need to understand the precise functioning of the tools. It is possible for them thanks to their own mathematical "culture" (or background) and also their will to investigate by themselves some new concepts. I interpret this as the need for "complete" praxeologies (Bosch, Fonseca & Gascón, 2004): the engineers do not only need the *praxis* (basically taught in the engineers schools), but also the *logos* (essentially depending on the Preparatory Cycle training).

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