A Characterisation of Attitude Grounded on Proportionality

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In Mathematics Education, the TMA that characterises the attitude towards mathematics has been reported. Under the assumption that mathematical knowledge plays an important role in student attitudes, we decided to characterise the attitude towards proportionality using the Three-dimensional Model for Attitude (TMA) as a preliminary model. To study the attitude, we use socioepistemological situations centred in proportionality, with three types of tasks: mixture, scale, and ratio and proportion. The data analysis showed a redefinition of the TMA's dimensions: the student's self-efficacy used to address the learning situation, the emotions triggered by working with the learning situation and the vision of working with a learning situation. Regarding proportionality, it is concluded that the dimensions that characterise the attitude depend on the task type and the learning situation design.

Keywords: Attitude, proportionality, mathematics, Socioepistemology.

Introduction

The Socioepistemological theory deals with education, i.e. the activities that accompany learning, and seek to improve teaching methods, the problems that provide the setting for the oral transmission of knowledge, cognitive processes, motivation and creation of positive attitudes (Cantoral & Farfán, 2003, p. 266). From this perspective we proposed a study of attitude.

The review that we ran on the history of research on attitude led to us to realise that the object of attitude has been school mathematical knowledge, i.e., when the students' attitude is investigated it is done in reference to the predispositions they have on their courses or when solving math problems in the classroom. From the theoretical perspective adopted, our contribution of research in the field of attitude was to question the role of mathematical knowledge when considered an object of attitude. For this reason we decided to focus on proportionality and to respond the research question: what attitudes are manifested when students solve learning situations focused on proportionality? We choose this mathematical knowledge because is transverse to the basic levels of Mexican education.

For many years there has been a large body of research on proportional reasoning. It seems that investigating the factors that influence the solution of problems proportional to students (Lamon, 2007) has passed to encouraging their command (Lamon, 1993; Howe, Nunes, & Bryant, 2010). Having a large amount of research on proportional reasoning in the field of Mathematics Education represented an advantage for us because in some of them, proportional task designs are shown. Some of the Learning Situations designs were retrieved from literature, not in order to replicate them, but with the purpose of investigating a variable that has not been considered in them: the attitudes of students towards proportionality.

Methodology

We adopted the TMA (Di Martino & Zan, 2010). In this characterisation, the attitude is made up of three dimensions that are recognised as the student's *emotional disposition, vision of mathematics and perceived competence.* As a hypothesis we consider that this characterisation would remain when particularising in a mathematical knowledge as an object of attitude, but we assumed that the properties of its dimensions would be particular of proportionality. The reformulation of the TMA that we obtain is shown in Figure 1.

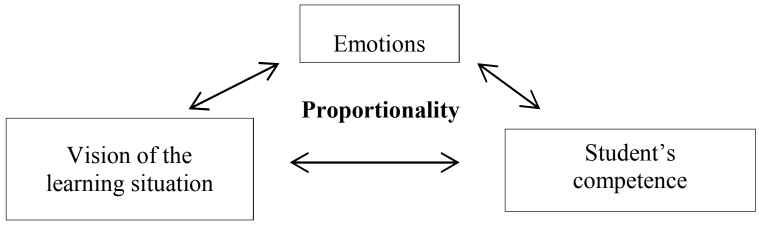

Figure 1. Attitude characterisation (Reformulation of Di Martino & Zan, 2010)

In order to study the attitudes towards proportionality, Learning Situations (LS) were designed. A socioepistemological LS is that problematic situation which allows us to favour the development of the learning process. The most important feature of LS is that they should not communicate to the student the knowledge that is expected to be assimilated, and the school background with which the student is expected to face the LS must be considered. The LS privilege the diversity of argumentations, and mathematics is considered the tool for decision making. The students' answers and argumentations are considered valid by the principle of contextualised rationality. They depend on their interpretation, the epistemological relativism and the encouragement of a redefinition of the students' previous knowledge. Some results of the problematisation of proportionality, specific research findings on proportionality and different frameworks where the LS are contextualised were considered in the design of our LS such as cooking, crafts, puzzles assembling, sale, art, anthropometry and photocopies. In total 10 LS were considered: 4 mixture tasks, 3 scale tasks and 3 ratio and proportion tasks. The LS contemplated practical activities, i.e. with the use of malleable material (M) and activities with pen and paper (PP).

Each type of task had specific goals. The mixture tasks were based on decision making; it was intended that the students decided which of two proposals was the best mixture. To achieve this they would have to make use of their qualitative proportional thought, ratios with integers and non-integers were considered. The scale tasks were focused on the use of the constant of proportionality. The students worked with exercises of successive application of the constant of proportionality in the reduction and secondly, of successive application of the constant of proportionality in the reduction and enlargement simultaneously, in these, an additive or multiplicative proportional thinking development was expected. The constant of proportionality was sometimes an integer and other times a non-integer. The ratio and proportion tasks were focused on the recognition of ratio and proportion in different situations. An example is the head-body ratio: over time the head size decreases compared to the size of the body, which increases; another example is the canon, a measurement used by the Greeks for sculpture.

Participants and context

In order to get to know the students' attitudes, a workshop entitled "Working with Learning Situations" attended by 20 students (10 female and 10 male, 14-15 years old) from a secondary school in Mexico City was developed. The workshop was held at the Mathematics Education Department premises, at CINVESTAV-IPN, it lasted 10 sessions of 1.5 hours. In each session the designed Learning Situations were worked. These sessions were filmed prior student and parent consent; all this in order to observe their reactions whilst working with the LS. The workshop sought to observe the closest relationship between the student and the proportionality, for that reason the LS were not resolved in a classroom, because it is regulated and organises the students' actions, influencing students' attitudes (García & Farfán, 2015).

In some cases, the students were interviewed individually in order to question them about working with the LS, and all were interviewed to learn about academic, family and personal matters such as school life, relationships with parents, and life outside school. The latter so as to get a triangulation of the information obtained by working with LS.

Data analysis

From our reformulation we bring three components of the attitude: *Emotions whilst solving the learning situation, Vision of the learning situation* and *Student's competence in the learning situation*. We used the Grounded Theory as a method of analysis, which provides a set of flexible strategies for collecting and analysing qualitative data. With these, theoretical categories directly based on data can be created, by constant comparison of the same data. Out of all the students' productions, we took 5 of each situation, corresponding to 5 students (3 males, M1, M3, M4 and 2 female, F2, F5) who were the most consistent throughout the workshop. Following the Grounded Theory guidelines, three phases in the analysis were carried out: open coding, axial coding and selective coding.

Grounded Theory is a methodology for developing theory that is grounded in data systematically gathered and analysed. This theory provides an analytic approach grounded on constant comparative method. In this methodology, theory may be generated initially from the data, or, if existing (grounded) theories seem appropriate to the area of investigation, then these may be elaborated and modified as incoming data are meticulously played against them. Researchers can also usefully carry into current studies any theory based on their previous research, providing it seems relevant to these but again the matching of theory against data must be rigorously carried out (Strauss & Corbin, 1994). In our case, we have adopted a previous model of attitude, which comes from coding using Grounded Theory, we sought to analyse up to what extent this helped us characterise the attitudes of students towards proportionality.

Taking as a reference the characterisation of pre-established attitude, we implemented three coding types (Grbich, 2013) with the evidence: 1) Open coding, involves word-by-word, line-by-line analysis questioning the data in order to identify concepts and categories which can then be dimensionalised. For this, we were labelling with words (codes) that which in evidence showed emotions, vision of the situation or the student's competence. Table 1 shows an intervention by the first author of this paper with student F2 at LS2, about mixing water and orange juice, she had said that the best mixture was that which had more juice and less water. In LS2 it was the mixture of hibiscus flower syrup and water, this time she said that the best mixture should follow the ratio of less syrup and more water, because the syrup is very concentrated and should be diluted with lots of water, contrary to the orange juice, that requires only a little water. When she was asked about the difference between her responses, we found evidence of emotion ‘to be happy’, because she managed to answer what was asked. See Table 1.

<u>Emotion</u>	**Properties**	**Data evidence**
To be happy	Having the answer (a good mixture depends on the taste)	[About her answer on the good mixture in LS1_M and LS2_PP] María: Last class you said that *we had to add more juice and less water, now you say that the best mixture comes from putting more syrup and less water* F2: *But it was orange juice*, Bonafina’s orange [orange juice] *is less strong compared with the taste of the concentrate. That’s why I said that.* María: So, on what does the best mixture depend? F2: On the taste that it has [**smiles**], *the best suggestion is the one that tastes the best,* it should taste of hibiscus [syrup], but not too strong [qualitative thinking], because if not, it won’t be a great tasting water, you have to pour in a lot of water to dilute it.

Table 1. Example of Open Codification. Mixture Task

2) Axial coding, involves you taking one category which has emerged in open coding and linking it.

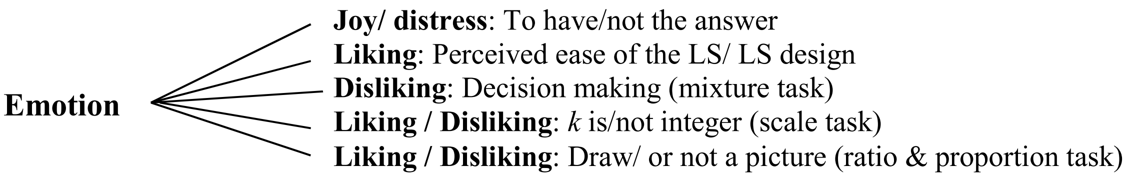

Figure 2. Emotion: axial coding

3) Selective coding involves you in the final putting together of two major files, the empirical data and the theoretical memos, both of which have been building up along parallel lines and should now be ready to put together within the conceptual sets which have been identified. We pretended to give objectivity to our analysis by integrating literature, according to the principles of Grounded Theory, literature can be used to confirm findings and, on the contrary, the results can be used to illustrate where literature is incorrect or too simplistic, or to partially explain phenomena. Bringing literature to writing demonstrates not only erudition, but allows knowledge of the area to extend, validate and refine (Strauss & Corbin, 2002). The properties identified in coding and the processes encouraged us to make confrontations with which in literature on affective domain noted, we focus

on emotions, the student's competence and vision of the situation, in order to give explanations of the attitudes that we find in the analysis.

Out of the TMA's three dimensions, two were kept, the emotion and vision of the learning situation dimensions, the competence dimension changed to self-efficacy, because with this term we identify not only whether the student was or was not competent, but also what motivated him to solve learning situations. Regarding mathematical knowledge the meanings associated with the tasks became evident, the procedures used, and the practices associated with each type of task. At this stage we refine the categories obtaining a single type of attitude, the proactive attitude, characterised by three components, as shown in Figure 3.

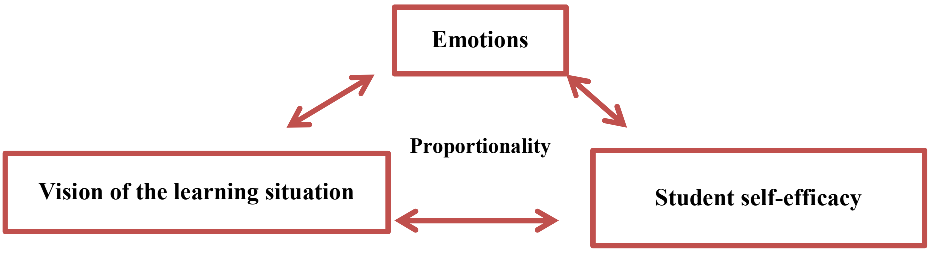

Figure 3. Characterisation of the proactive attitude

Results

The data analysis showed a single type of attitude, which we have called proactive, because students showed willingness to solve the LS proposed; in addition, they showed initiative in developing these in order to solve them. Proactivity was shown diversely amongst the students, the reason being the personality of each student and the type of proportional task faced. The results confirmed our hypothesis, the attitude towards proportionality keeps the dimensions of the attitude towards school mathematical knowledge, but the subcategories of each of these are specific to the student interaction with the proportionality and to the interaction design of the learning situation. These findings led us to rename the categories of the original model as follows: 1) emotions, 2) vision of the learning situation and 3) student self-efficacy (see Table 2).

The emotions that we identified are presented dichotomously: joy and distress, liking and disliking. Each pair appears when the triggering situations oppositely occur, for example, when the answer to the LS is known joy appears, otherwise distress appears; liking and disliking are triggered by specific characteristics of the task, for example, in the mixture tasks, decision making triggers disliking in some students; in the scale tasks, the disliking is triggered by working with a non-integer constant of proportionality and liking when working with an integer constant of proportionality. Literature has reported that the use of non-integers, even ½, causes difficulty to students, they became more easily frustrated with these problems (Lamon, 1993).

Emotion	Vision of LS	Self-efficacy
Working with the LS	Relational	Relative to knowledge
	Familiar context of the LS	**Meanings**: Relation ratio/comparison
Joy: Having the answer	Manipulation of the variables that are related	**Procedures**:
		Mixtures: Testing of the mixture, preparing the mixture by trial and error
Distress: not having an answer		
	Understanding the LS	Scales: Manipulation of the factor of proportionality in the measurements of a puzzle
Liking: Perceived ease of the LS/ LS design	Instrumental	Ratio and proportion: Times that a variable can be contained by the other
Regarding knowledge	Mechanical calculations	**Argumentations**: Comparing the variables; taste of the mixture, colour of the mixture
		Associated practices: Comparing, building a unit of measurement
Disliking: Decision making (Mixture)	The LS is not understood	
		Personal
		M1 subsequent to working with a LS_PP
Liking / Disliking: *k* is/not an integer (Scales)		It was very exciting because I was right and **had to convince M4** of the correct answer {vicarious experience}. The exercise is similar to those at school {mastery experience}, but **being a picture** I thought it was easy {self-efficacy beliefs} as **I mix colours a lot because I like drawing and painting** {contextualised rationality}.
Liking / Disliking: Draw a picture (Ratio and proportion)		

Table 2. Proactive attitude towards proportionality

The data coding of the competence category answered the questions: How and why each student solved the LS, because of that, it became necessary to enlarge it and we did so with the concept of self-efficacy. According to Bandura it refers to people´s beliefs about their capabilities to exercise control over events that affect their lives and their beliefs in their capabilities to mobilise the motivation, cognitive resources, and courses of action needed to exercise control over task demands (Usher & Pajares, 2009). There are four self-efficacy sources: 1) *mastery experience*, the most powerful of which is the interpreted result of one's own previous attainments; 2) *vicarious experience* are gained by observing others perform tasks, students compare themselves to particular individuals such as classmates, peers as they make judgments about their own academic capabilities; 3) *social persuasions* refers messages from others about one's ability to accomplish a task and 4) *emotional and physiological states*, such as anxiety, stress, fatigue, and mood success or failure to the degree that they feel similar to the model in the area in question. We identified the latter source in the category of emotions.

The self-efficacy category is divided into two types: relative to knowledge, where the meanings associated with proportionality and ratio are reported, the argumentations and procedures used by the students in each of the LS. Moreover, we considered the category relative to personal matters, we identified in it the self-efficacy through the sources described above.

The vision of the LS was kept from the TMA, but its properties changed, the relational vision was associated with several factors, including the understanding of the LS, the familiarity of the context and manipulation of variables in the ratio of proportions. These factors favoured a relational vision in the students that was associated with solving the LS through argument the answer, however instrumental vision only triggered when students got an answer without argument.

Discussion and conclusions

We found that some designs of the LS provoked an attraction in the students that favoured a proactive attitude. It was the case of LS where practical activities were included such as preparing recipes and solving puzzle, because those represented a tangible instrument for the students that allowed them to validate the work carried out. Regarding the type of task, we found that mixture tasks were more interesting for students for two reasons, decision making and manipulation of the variables in the proportional ratio, by mixing liquids or preparing flavoured water. This fact evidences attitudes toward work with the learning situations that we can call "attitude toward mathematics" such as the liking of responding to a question or working with a manipulative material such as mixture of liquids, these results are according with those reported in the literature about affect and problem solving (Grootenboer & Marshman, 2016). It seems that attitudes toward working with mathematics appear when it is particularized in mathematical knowledge and this is understandable because of the nature of mathematics and classroom norms, for example having to answer questions or to solve problems. What our research shows is that certain characteristics of mathematical knowledge are capable of triggering attitudes, for example scale tasks were rated less favourably, in some cases, due to the proportionality factor used, for them it was difficult to manipulate a k=0.75 and to reduce with it the measures of the puzzle, but that was not the case with k=0.5, this one was easier.

Concerning student characteristics their personal beliefs towards mathematics and judgments of self-efficacy influenced their work with the proportional, the relations that the students were establishing over the duration of the workshop with their peers also intervened, in some cases these relations were decisive to resolve the required learning situation. But the main reason that triggered this proactive attitude was the attendees' objective: to improve their math skills. This goal was shared by the participants and their mothers, who were alongside them, taking them to each of the workshop sessions. This goal was the motivation that sustained the proactive attitude of the students. This result is consistent with those found by Gomez-Chacon (2013) who suggests that affect is related to motivation through goals and self-concept.

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