An analysis of freshmen engineering students' notes during a preparatory mathematics course

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I analyse the notes of a group of first year engineering students who attended a course in pre-calculus mathematics. Being interested in verbalisation skills at the beginning of university, I adopt a narrative lens to analyse the notes: I see a lecture as a story being told by the teacher and the students' notes as re-tellings of the teacher's story. I focus on the way the students condense the mathematical content in their written notes in two distinct teaching formats: a traditional frontal lesson and the concluding phase of a classroom discussion after a small group activity.

Keywords: Secondary-to-tertiary education transition, students' notes, narrative approach.

Introduction

This paper is part of a wider research project on freshmen engineering students at the Polytechnic of Milan, aimed at understanding their difficulties during the first year of their studies. Recent research have found that mathematics at tertiary level is difficult for engineering students (see e.g. Gomez-Chacon, Griese, Rosken-Winter & Gonzales-Guillen, 2015). Gueudet (2008) provides a detailed overview of mathematics education studies concerning the transition from school to university and she identifies the theme that is the core of my research, namely: the *students' organisation of knowledge that is new to them* at the beginning of university.

Boesen, Lithner & Palm (2010) argue that the kind of task assigned to the students affects their learning: tasks with low levels of cognitive demand lead to rote-learning by students and, consequently, their inability to solve problems that are unfamiliar to them (for instance, the ones that require conceptual understanding). Breen, O'Shea and Pfeiffer (2013) define an 'unfamiliar task' as a task "for which students have no algorithm, well-reharsed procedure or previously demonstrated process to follow" (p. 2318), and provide evidence that this kind of tasks raises an awareness about the need for more than procedural understanding of mathematics, thus easing the students' transitions to university math practices. Also the teaching strategies employed in the class can influence the development of one type of knowledge more than another: teacher-centred methods would favour the development of procedural knowledge and student-centred methods would favour conceptual knowledge (see e.g. Garner & Garner, 2001; Allen, Kwon & Rasmussen, 2005). Inspired by these studies, I investigate how the students organise the knowledge in their notes during a preparatory math course.

To take notes does not involve only mathematical ability. It involves also verbalisation skills. O'Neill, Pearce and Pick (2004) found that there is a correlation between performance in generating *narratives* and *mathematical ability* as early as in primary school years. With Nardi (2011), I recognise the centrality of the students' ability to use ordinary language to construct and convey mathematical meaning and I investigate undergraduate engineering students' notes in a preliminary math course at their first year at university. With Nardi (2011), I value the students' attempts to mediate the mathematical meanings through words, symbols and diagrams and I maintain that at the basis of the students' difficulties in dealing with a discursive shift from secondary to university mathematics there are: (a) undervalued verbalization and (b) premature compression. According to the former, "the students undervalue, and often avoid entirely, expressing their mathematical thoughts verbally" (Nardi, 2011, p.2056); according to the latter, "students' mathematical writing is typically prematurely compressed, namely ridden with gaps, leaps and omissions" (ibid.).

Theoretical framework

Andrà (2013) examines the relationships between a teacher's lecture and the students' notes by viewing the lecture as a kind of story that the teacher tells and the students' notes as retellings of the teacher's story. A mathematical lesson seen as a *story* can be analysed in terms of its components: its characters, setting, action, plot, and moral (see also Bal, 2009).

Mathematical objects, in fact, can be considered the *mathematical characters* of a story (Dietiker, 2012). They can play a central or a peripheral role, have multiple names, and have properties that can be introduced and developed gradually. The *setting* is the space where characters are placed. Sometimes the setting is not obvious, as it refers to underlying assumptions and/or axioms. The setting may also involve different registers, such as algebra or the Cartesian coordinate system. The *action* is that which the actor performs. In mathematical stories, the result of an action can be a change in an object or in a setting, or both. According to Dietiker (2012), we see that, unlike in literary stories, mathematical ones can change actions into objects (through reification). Andrà (2013) observes that the students miss important (teacher's) mathematical actions in their notes and more in general Morgan (1998) notices a relative absence of active verbs in mathematical writing. The *moral* can be seen as the intended message of the lesson. The *plot* is the sequence of actions and it involves the *shaping* of the story, which is linked to its *aesthetic effects*: for instance, the rhythm and the frequency of the story (Bal, 2009) may affect the students' focusing on the areas of emphasis of the story and foster his anticipative acts. Some moves might displace attention away from what the teacher wants to communicate: for example, repeating the name of a character often may lead the students to think that the actual name is important, or more important than its properties. Other moves may induce the students to believe that the setting is unimportant. These moves can be interpreted in terms of Rotman's (1988) schema, which distinguishes between invitations for the reader to be a "thinker" and ones that prompt the reader to be a mere "scribbler". This distinction provokes a further distinction, namely: to think about note-taking as mere consumption of mathematical meaning or to think about it as active production of meaning. It is possible to interpret Rotman's schema with respect to the students' notes in this way: a scribbler is a student who reports mainly the mathematical characters of the story and misses the actions, so that the notes result to be compressed and ridden with gaps, leaps and omissions (see also Nardi, 2011). A scribbler is also a student who avoids to put her thoughts in her notes, and limits herself to copy and/or report what the lecturer is saying/writing. The plot is the same plot of the story told by the lecturer. A thinker, instead, re-organises the content of the lesson, she (re)structures the plot so that it becomes accessible to herself even after the lesson ends. A thinker pays attention to the details and also records the mathematical actions so that her notes are not overly compressed and under-verbalised. In Andrà's (2013) understanding of Rotman's schema, furthermore, some moves invite the students to be thinker while other ones to be scribbler. In view of Boesen, Lithner & Palm's (2010) findings, we consider two scenarios: in the first one, the lecturer proposes a group activity on a conceptual and unfamiliar task and the students' notes are taken during the classroom discussion that follow the group activity; in the second scenario, the lecturer assigns a procedural task and corrects it on the blackboard. The interest is to see how the students' notes change (if so) in the two different scenarios.

Methodology

The Polytechnic of Milan, like many universities all around the world, organizes some courses before the beginning of the first semester, which have the purpose to recapitulate the basic knowledge that is necessary for the students to successfully attend the courses at the first academic year. One of these preparatory courses is on pre-calculus mathematics. Since three years, the course is organised according to a flipped classroom pedagogy: the students (are supposed to) watch a series of videos in a MOOC and at university the lecturers of the preparatory course involve them in groupwork activities aimed at deepening their understanding of the basic math concepts and expose the students to frontal lessons with routine exercises. The mixed method of teaching serves the purpose of both exposing the students to a new, "conceptual" teaching and to make them feel comfortable with teaching practices that are more typical of secondary school.

During the first lesson of the preparatory course, among the exercises given about polynomials, one had a conceptual nature, since it said "The polynomial p(x) is divisible by the polynomial q(x) if…". This is a kind of task that is unfamiliar for Italian students, since it asks to reflect about the definitions and the students do not have a well-established procedure to resort to. It was firstly dealt with in small groups, then discussed at classroom level. During this last phase, the teacher wrote the steps of the solution at the blackboard and the students took notes. Another task had a procedural nature and was not unfamiliar for the students: two polynomials were given, p(x) and q(x), and the students were asked to divide p(x) by q(x). It was solved at the blackboard by the teacher. I compare and contrast the students' notes in these two different situations: one familiar and procedural (i.e., linked to actions), one unfamiliar and conceptual (i.e., linked to characters).

I collected the notes taken by 10 students chosen at random and I selected 4 of them to be analysed in this paper since they are contrasting. The students are identified with four fictitious names: Angela, Filippo, Roberto and Vincenzo. Their notes are analysed through a narrative lens, identifying: the mathematical characters; their setting; the mathematical actions, understood in terms of operations made on/by the mathematical characters; the plot, or the organisation of the content on the sheet of paper; the moral. The four students are inferred to be scribbler or thinker by looking at these elements.

The research questions read as follows: (a) how do students organise their notes? (b) which elements of the teacher's "story" are recorded, and which ones are discarded? (c) in which cases the students are scribblers and in which ones are they thinkers?

Data analysis

Figures 1-8 report the four students' notes regarding the two tasks. Since they are in Italian, a translation is provided in the caption of each figure.

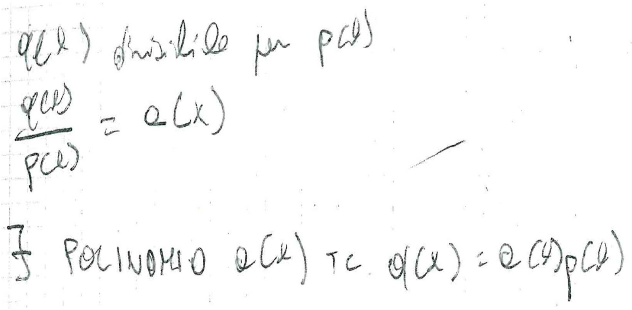

Figure 1: Vincenzo's notes about the "conceptual task". On the first row he writes "q(x) divisible by p(x)". On the second row he writes a formula and in the last row: " polynomial a(x) s.t. q(x) = a(x)p(x)".

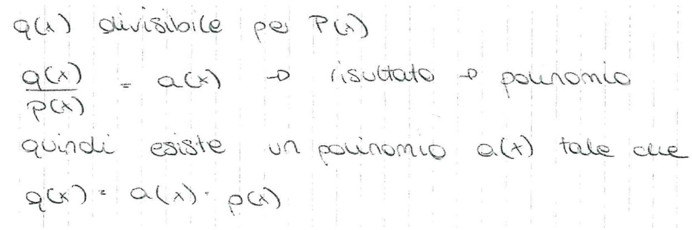

Figure 2: Angela's notes about the "conceptual task". On the first row she writes "q(x) divisible by p(x)". On the second row she writes the ratio, then she adds an arrow and writes "result", then another arrow and "polynomial". On the third row she writes "hence there exists a polynomial a(x) such that q(x) = a(x)p(x)".

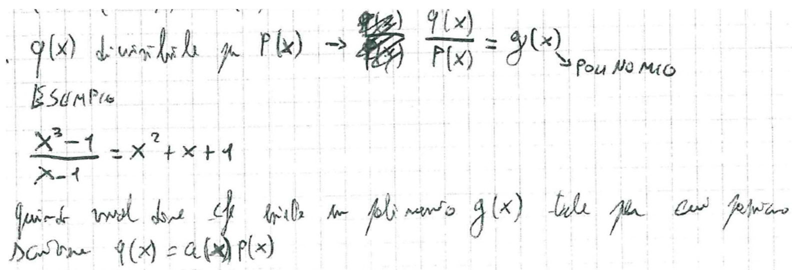

Figure 3: Filippo's notes about the "conceptual task". On the first row he writes "q(x) divisible by p(x)", then he draws an arrow and writes a ratio, from which he draws another arrows and writes "polynomial". On the second row, he writes "example" and in the third row he writes a formula. In the last row: "hence it means that there exist a polynomial g(x) such that I can write q(x) = a(x)p(x)".

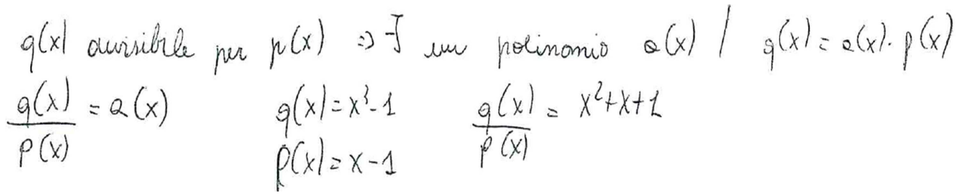

Figure 4: Roberto's notes about the "conceptual task". On the first row he writes "q(x) divisible by p(x)", then he draws an arrow and writes " a polynomial a(x) | q(x) = a(x)p(x)".

From Vincenzo's notes (Figure 1), we can see that the story has: (A) two *mathematical characters*, p(x) and q(x) and there's a relationship between them, being the latter divisible by the former; (B) the *mathematical action* is a division; (C) the *moral* is that there exists a polynomial a(x) such that q(x) = a(x)p(x). Vincenzo's plot is linear: each row is put below the other one, with no connections.

Also the plot of Angela's story (Figure 2) is linear: (A) is followed by (B) that is followed by (C). But she also adds an arrow on the right side of (B) and she writes "result", then another arrow and then "polynomial". For her it is worth noticing that the result of the mathematical action is a polynomial (which is a(x), a new character), namely a character that still has the same properties of the other two. While the setting was implicit in Vincenzo's notes, it emerges in Angela's ones: the setting is the set of polynomials. Instead of the mathematical symbol for "there exists", she writes in words and she also adds "hence" at the beginning of (C): the moral is made more explicit and the verbalisation is less condensed with respect to Vincenzo.

Filippo (Figure 3) organises the texture in a non-linear way: he writes (A) and on the same row he writes (B), to which he draws an arrow and writes "polynomial". Like Angela, also Filippo remarks this detail. Vincenzo and Angela do not write the example, while Filippo does. Hence, a story in the story is told: it's the story of the two characters that become two particular polynomials. At the end of the story, Filippo writes (C) in a fashion that is similar to Angela's one.

Filippo's story is slightly less linear than the stories re-told by Angela and Vincenzo, but the student that writes a different plot is Roberto (Figure 4): he writes (A), an arrow, then (C) on the first row, namely he puts at the first line the characters and the moral, then on the second row he writes the action, which is (B), and the story in the story, namely the example.

We can infer that Roberto is a thinker, since he re-organizes the knowledge, while Vincenzo is a scribbler, since he reports the story in a linear way. Also Angela and Filippo act as scribblers: in a sense, we can say that they are more accurate than Vincenzo, since they report more details, but do not re-organise the content of the lesson as Roberto does. Roberto, in fact, does not only remark what is worth noticing, he establishes a hierarchy in the mathematical content: characters and moral on the same, first row, and the action plus the example on the same, second row.

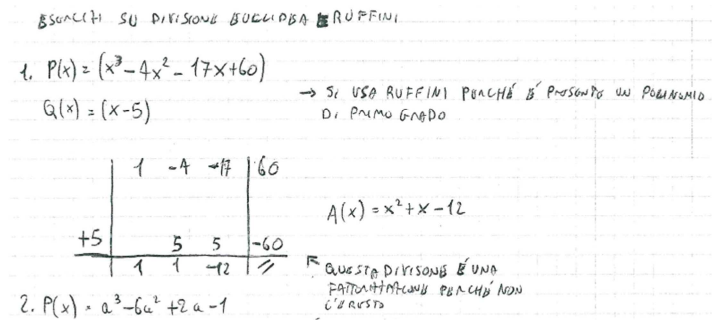

Figure 5: Filippo's notes about the "procedural task". On the first row he writes "Exercises on euclidean division and Ruffini's division". To the right of the first arrow he writes "Ruffini is used because a first order polynomial is present". To the right of the arrow pointing to -60, he writes "this division is a factorisation, because there's no reminder".

If we look at the "procedural task" in Filippo's notes (Figure 5), we notice that he employs a more linear structure with respect to the "conceptual" one. He writes: (0) the title of the story ("Exercises on the Euclidean division and Ruffini"), then (1) he presents the characters P(x) and Q(x), then (2) the series of actions in the Ruffini's grid. The new character, (3) A(x), the result of the actions, is present to the right of the grid. At the right side of the paper he adds comments that are connected to the "story" by means of arrows: such comments better characterise, and justify, the actions that are made. Like in the conceptual case, we see them as details that are worth to be noticed by Filippo.

Like in the conceptual case, we can say that Filippo is an *accurate scribbler*. Also Vincenzo's notes (Figure 6) have a linear structure with no connections between (1) and (2), or between (2) and (3). Also Vincenzo adds the comment "we use Ruffini when we divide by a order-1 polynomial", but this comment about the actions is put below the characters with no arrow.

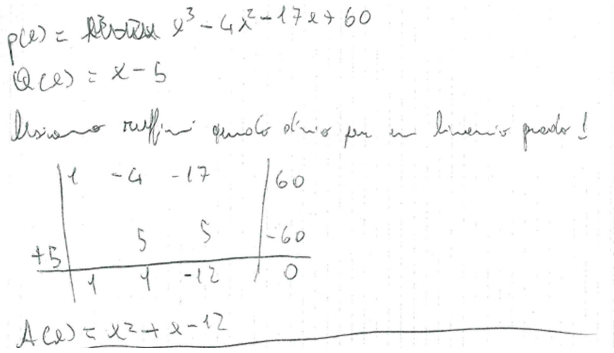

Figure 6: Vincenzo's notes about the "procedural task". The first sentence reads "We use Ruffini when we divide by an order-1 polynomial".

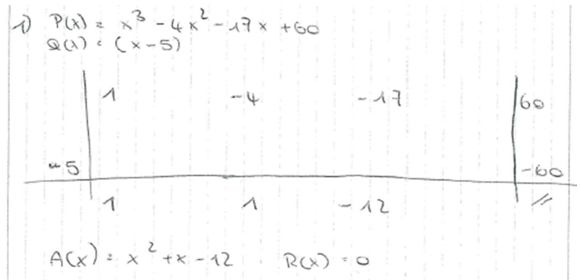

Figure 7: Angela's notes about the "procedural task".

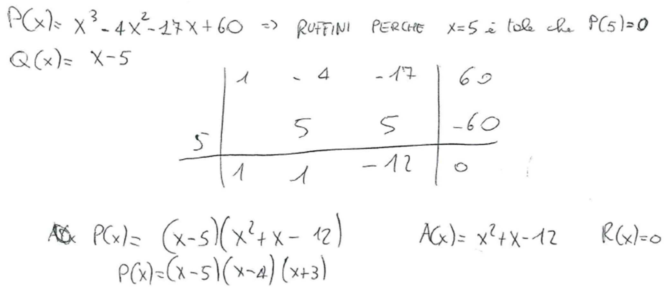

Figure 8: Roberto's notes about the "procedural task".

Angela's notes (Figure 7) have no words, just symbols: we can say that there's only one register present, the symbolic one. She records the characters, i.e. (1), the actions, i.e. (2), and the new character that results from the action, i.e. (3). Differently from Filippo, whose notes have a rather linear structure, Angela's ones are even more linear and essential, as if she wants to record just the essential facts. Like Vincenzo, Angela is an (inaccurate) scribbler.

Roberto (Figure 8) records (1), then (2), then (5)-(3)-(4) on the same line. To the right of (1) he remarks "Ruffini, because x=5 is such that P(5)=0", hence noticing a detail that is different from the ones recorded by Filippo and Vincenzo and that is less general than those ones: Ruffini's algorithm can be used for any value of x when q(x) is an order-1 polynomial, not only for those polynomials where the value of x is a zero. Since the lecturer has said something different (see Filippo's or Vincenzo's notes), we can infer that Roberto added a detail that generated from his own knowledge about polynomials. As for the conceptual task, Roberto reorganises the space of the sheet and we can infer that he acted as a thinker.

Discussion

We discuss the data analysis in terms of what the distinction between scribblers and thinkers can add to: (1) our knowledge of the students' verbalisation skills (answering to the question *which elements of the teacher's "story" are recorded, and which ones are discarded?*); (2) our understanding of how the students organise their knowledge (*how do students organise their notes?*); (3) whether the teaching strategies have an influence on conceptual and procedural understanding of mathematics (*when do the students act as scribblers and when as thinkers?*).

Comparing Angela's and Vincenzo's notes, we see that both them act as scribblers in both tasks. We can further infer that Angela is a scribbler because the course is recapitulating mathematical concepts that are familiar for her, hence she does not need to put so many details in her notes. We infer this also by looking at her notes for the conceptual task: we commented that, in that case, she had time to record details that are worth to be noticed and she didn't give us the impression that she was rushing to keep the pace of the lecturer. For Vincenzo, it is a completely different story: he is struggling to remark all that is relevant, since the lesson is difficult for him. A conclusion that we can draw is that a student acts as a scribbler in two cases: either if the mathematical content is too easy for her, or if it is too hard.

Nardi (2011) pointed out that the students under-verbalise and iper-condense the mathematical discourses. As regards the conceptual task, we can see that Vincenzo and Roberto condense the mathematical content more that Filippo and Angela, but Roberto does it in a completely different way compared to Vincenzo: Roberto compresses and reorganises the content, to have the character and the moral on the same row, and the actions plus the example on the second row, while Vincenzo linearly puts the elements of the story one after the other. Andrà (2010) analysed the teaching styles of university lecturers and she concluded that in a blackboard modality (namely, when the lecturer is mostly writing on the blackboard) the students have to adjust the pace of their note-taking to the pace of the lectures' writing. By comparing Vincenzo's and Roberto's notes, indeed, we can imagine the former making an effort in dealing with a pace that is too fast for him, to the point that he does not have time to record the details that Angela and Filippo remarked, while Roberto stops and thinks (fast) where he wants to put what is told by the teacher.

Angela and Filippo are accurate scribblers, hence they tend not to iper-condense the math content. As well, Angela and Filippo tend not to under-verbalise when they take notes on the conceptual task. Why are some students more accurate scribblers than others? Andrà (2010) interpreted this difference in terms of each student's ability to keep the pace of the lesson at the blackboard, but I would also add that it depends on the student's views: for some students it seems necessary to record all the possible details, while for others it seems a question of being short.

Looking closely to Angela's notes, and comparing her notes on the conceptual task and on the procedural one, we can see a difference: in the first case, she adds details and comments that she discards in the second case. Vincenzo does not add comments in neither case, and Filippo accurately adds details in both cases, hence Angela is the student on which the procedural vs conceptual nature of the task provokes different modalities of taking notes, and actually the conceptual nature of the task invites her to remark more details. This has an impact on her verbalisation skills and on conceptual reflection.

References

Allen, K., Kwon, O. N. & Rasmussen, C. (2005). Students‘ retention of mathematical knowledge and skills in differential equations. *School Science and Mathematics, 105*(5), 227-239.

Andrà, C. (2010). Teaching styles in the classroom: how do students perceive them? In: M.M.F. Pinto & T.F. Kawasaki (Eds.), *Proceedings of the XXXIV Conference of the Psychology of Mathematics Education* (PME 34), Belo Horizonte, BR: PME (Vol. 2, pp. 145-152).

Andrà, C. (2013). How do students understand mathematical lectures? Note-taking as retelling of the teacher's story, *For the learning of mathematics, 33*(2), 18-23.

Bal, M. (2009). *Narratology: Introduction to the theory of narrative*. (C. Van Boheemen, Tran.) (3rd ed.). Toronto: University of Toronto Press.

Boesen, J., Lithner, J, & Palm, T. (2010). The relation between types of assessment tasks and the mathematical reasoning students use. *Educational Studies in Mathematics, 75*, 89-105.

Breen, S., O'Shea, A., & Pfeiffer, K. (2013). The use of unfamiliar tasks in first year calculus courses to aid the transition from school to university mathematics. In: *Proceedings of the 8th Congress of the European society for Research of Mathematics Education* (2316-2325).

Dietiker, L. (2013). Mathematics texts as narrative: Rethinking curriculum. *For the Learning of Mathematics, 33*(3), 14–19.

Garner, B. & Garner, L. (2001). Retention of concepts and skills in traditional and reformed applied calculus. Mathematics Education Research Journal, 13(3), 165-184.

Gómez-Chacón, IM, Griese, B, Rösken-Winter, B, & Gonzàlez-Guillén, C. (2015). Engineering students in Spain and Germany—varying and uniform learning strategies. In: N. Vondrova & K. Krainer (Eds.), *Proceedings of the 9th conference of european researchers in mathematic education*.

Gueudet, G. (2008). Investigating the secondary-tertiary transition. *Educational Studies in Mathematics, 67*, 237-254.

Morgan, C. (1998). *Writing mathematically. The discourse of investigation.* London, UK: Falmer.

Nardi, E. (2011). 'Driving noticing' yet 'risking precision': university mathematicians' pedagogical perspectives on verbalization in mathematics. In: *Proceedings of the 7th Congress of the European society for Research of Mathematics Education* (2053-2062)

O'Neill, D. K., Pearce, M. J., & Pick, J. L. (2004) Predictive relations between aspects of preschool children's narratives and performance on the Peabody Individualized Achievement Test - Revised: Evidence of a relation between early narrative and later mathematical ability. *First Language, 24,* 149-183.

Rotman, B. (1988). Toward a semiotics of mathematics. *Semiotica, 72*(1/2), 1-35