An Investigation of Middle School Mathematics Teachers’ Knowledge for Teaching Algebra

Nurbanu Yılmaz\(^1\) and Ayhan Kürşat Erbaş\(^2\)

\(^1\)Bülent Ecevit University, Faculty of Education, Zonguldak, Turkey; nurbanuyilmaz@beun.edu.tr
\(^2\)Middle East Technical University, Faculty of Education, Ankara, Turkey; erbas@metu.edu.tr

The purpose of this study was to investigate middle school mathematics teachers’ knowledge for teaching algebra. The participants of the study were 48 mathematics teachers from various middle schools. A questionnaire was conducted in order to collect data about the teachers’ knowledge related to teaching of algebra. The results showed that the participant teachers were competent in making transitions among different algebraic representations. However, they had difficulties in explaining the conceptual bases of some of the algebraic concepts and procedures. In addition, results indicated that some of the teachers had similar difficulties and misconceptions with students as they were depicted in the scenarios in the questionnaire.

Keywords: Pedagogical content knowledge, algebra, middle school mathematics teachers.

Introduction

Teachers’ knowledge is considered as one of the most important predictor of the student achievement (Hill, Rowan, & Ball, 2005). In recent years, therefore, researchers have focused on the professional knowledge of teachers (Ball, Thames, & Phelps, 2008; Grossman, 1990; Shulman, 1986, 1987). As Knowles, Plake, Robinson, and Mitschell (2001) stated, what teachers should know and be able to do are issues which continuously change and develop as values of the society come up with the changes. Therefore, teachers need different types of knowledge in order to fulfill those expectations. Subject matter knowledge (SMK), pedagogical knowledge, and pedagogical content knowledge (PCK) are the major components of teacher knowledge which are mentioned frequently in the related literature (Ball et al., 2008; Cochran, DeRuiter, & King, 1993; Magnusson, Krajcik, & Borko, 1999; Shulman, 1987). There is an agreement in the teacher education literature that strong subject matter knowledge is a central component of teacher competency (Krauss et al., 2008). However, merely having strong mathematics knowledge does not guarantee effective teaching (Ball et al., 2008; Kind 2009).

Effective teaching requires making the content accessible to students, interpreting the questions and productions of students, and being able to explain or represent ideas and procedures in multiple ways (Hill, Sleep, Jewis, & Ball, 2007). Teacher competency is defined as the cognitive ability in order to develop solutions for problems concerning teaching profession and applying these solutions in various situations successfully (Weinert, 2001). Providing meaningful and effective activities for students’ learning is considered essential for teacher competency (Knowles et al., 2001). Therefore, pedagogical content knowledge, “the most useful ways of representing and formulating the subject that makes it comprehensible to others” (Shulman, 1986, p. 9), is seen as a core component of teacher competency and an indispensable part of teacher knowledge base. A study conducted by Kind (2009) in which several PCK models were investigated revealed that representations and instructional strategies and students’ subject specific learning difficulties were considered as two core dimensions of PCK in most of the studies (e.g., Grossman, 1990;
Magnusson, Krajcik & Borko, 1999; Shulman, 1987). Therefore, among others, the components that are knowledge of students’ learning of mathematics and knowledge of teaching mathematics were chosen as the focus in this study.

In the literature, there have been several studies in relation with the pedagogical content knowledge of mathematics teachers concerning knowledge of algebra and teaching of algebraic concepts (e.g., see Doerr, 2004; Güler, 2014; McCrory, Floden, Ferrini-Mundy, Rackase, & Senk, 2012). As the Mathematics Study Panel (2003) indicates, the knowledge of algebra is an important component for the mathematics knowledge of students. Moreover, teaching of algebra especially in middle school is crucial since the algebra learnt there constitutes a basis for the high school and university mathematics (Mathematics Study Panel, 2003). Thus, it can be inferred that teachers’ knowledge of algebra and teaching of algebra in middle schools are worth studying since the professional knowledge of the teachers is one of the most important predictors of the achievement of students (Hill, Rowan, & Ball, 2005). Since the knowledge of teachers is an important factor for effective teaching, the assessment of teacher knowledge is an important step to understand the competency of teachers or the quality of teacher education programs. Although there are several studies based on the algebra knowledge of students, there is limited research based on the instruction of algebra (Güler, 2014; Kieran, 2007; Ladele, Ormond, & Hackling, 2014). Therefore, which knowledge component is required by the teachers and how this knowledge could be developed need to be investigated in order to improve the algebra instruction (Kieran, 2007). For this reason, there is a need for the theory building on what teachers need to know related to teaching of algebra and how this knowledge could be developed by teachers. In this study, SMK and PCK were considered as separate dimensions of teacher knowledge and PCK of middle school mathematics teachers was investigated for teaching algebra. As for the main theoretical framework of the study, we used “algebraic knowledge for teaching” which was adapted by Güler (2014) from the study of Ferrini-Mundy, Floden, McCrory, Burrill, and Sandow (2005). The framework is a three-dimensional model including three main components, namely algebra content, algebra knowledge for teaching, and domains of mathematical knowledge. Algebra content consists of two main categories, algebraic expressions, equalities, and inequalities and linear and non-linear functions and their properties. Algebra knowledge for teaching includes advanced algebra, knowledge about learning of students, and knowledge about representations of the content. Domains of mathematical knowledge include basic concepts and procedures, representations, applications, and reasoning and proof. In this study, algebraic knowledge for teaching dimension of the model was focused on. The only difference of adapted framework (Güler, 2014) from the original one (Ferrini-Mundy et al., 2005) was in algebraic knowledge for teaching dimension which includes school algebra, advanced algebra, and teaching knowledge components. The knowledge about learning of students, and knowledge about representation of the content segments of this dimension were investigated within the study in relation with algebra content and mathematical knowledge context dimensions.

The purpose of this study was to investigate the knowledge of middle school mathematics teachers in relation with teaching of algebra. In this scope, two components of pedagogical content knowledge of middle school mathematics teachers were investigated; knowledge of the learning of students and knowledge of teaching mathematics. Moreover, the difficulties and strengths of middle
school mathematics teachers were investigated in relation with the knowledge of teaching algebraic concepts. Therefore, the research questions that guided this study were as follows:

- What are the difficulties and strengths of middle school mathematics teachers in relation with the teaching of particular algebraic concepts?
- What is the pedagogical content knowledge of middle school mathematics teachers in relation with particular algebraic concepts?

**Method**

Participants of the study were 48 middle school mathematics teachers from different public and private schools in Turkey who participated in the study voluntarily. Their teaching experiences (in years) ranged from 1 year to 26 years ($\overline{X} = 8.0, SD = 5.3$). Developed by Güler (2014) based on literature, the instrument used in this study was a questionnaire comprising of multiple-choice items and open-ended items intended to assess teachers’ knowledge for teaching algebra at the middle school level. For the instrument, the scores were averaged across 20 items to control the reliability of the instrument ($\overline{X} = 17.8, SD = 5.9$). Also, Cronbach’s alpha was estimated as 0.81 for person reliability and 0.94 for item reliability. Moreover, in order to ensure validity of the instrument, item analyses were carried out and expert opinion was taken in order to show that the content of the instrument coincides with the conceptual framework. In the current study, the instrument was used with the permission of the developer (Güler, 2014).

There were totally 20 items in the questionnaire. In order to collect the data, participants were confronted with the critical situations including scenarios of different algebraic concepts. The algebra content tested in the instrument was in line with the mathematics curriculum for the middle school level (5-8th grades) in Turkey. The instrument consists of two components in relation with PCK, knowledge of students’ learning and knowledge about representation of the content. Moreover, the content of the instrument is constructed under two domains, mathematical knowledge content and algebra content. Mathematics content includes basic concepts and procedures, representations, applications, and reasoning and proof. Algebra content includes algebraic expressions, equality, and inequality, linear and non-linear functions and their properties.

The use of a questionnaire with a survey type design was preferred in order to collect data from a large sample of teachers. There was no time limitation for completing the questionnaire. Descriptive analyses and item based in-depth analyses were carried out in order to have a general overview on algebra related PCK of teachers. Analyses were conducted based on the rubric prepared by Güler (2014) for each item separately. Therefore, the frequencies and percentages of correct, particularly correct, or incorrect answers were calculated in order to investigate the performances of all teachers for each item. To illustrate, the answers to the 10th item (see Figure 1) were categorized as correct if the teachers explained why the inequality sign changed direction by using algebraic expressions, partially correct if the teacher explained it by using particular values of $x$, and incorrect if the answer was wrong, invalid, or missing.
In general, answers were coded as correct when the teacher provided correct answers/results to the questions, used algebraic expressions to explain an algebraic concept conceptually, and explained why the answer of the student was wrong in the scenario by giving underlying reasons. The answers coded as partially correct included the answers in which teachers gave inadequate explanations and used particular values to show the validity of an algebraic procedure. The answers coded as incorrect were the wrong, invalid, or missing ones. To ensure the reliability of coding, two independent scorers coded all of the open-ended items in the questionnaire for half of the participants \((n = 24)\). The interrater agreement across both scorers was high \((\text{percent agreement} = 92.80\%)\).

**Findings**

Nearly half of the items in the questionnaire were related to making transition among different algebraic representations. As results suggested, most of the teachers participated in this study were able to make the transition among the rhetoric, symbolic, and geometric representations of the algebraic expressions. Results also showed that most of the teachers answered the items correctly which required the identification of the algebraic problems for given algebraic expressions. The results regarding the 18th item in which "The difference between the equation and (algebraic) identity" was asked are presented in Table 1. According to results, most of the teachers answered the 18th item correctly.

<table>
<thead>
<tr>
<th>Item</th>
<th>Correct</th>
<th>Partially correct</th>
<th>Incorrect</th>
</tr>
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<tbody>
<tr>
<td>f</td>
<td>%</td>
<td>f</td>
<td>%</td>
</tr>
<tr>
<td>6th item</td>
<td>3</td>
<td>6.3</td>
<td>17</td>
</tr>
<tr>
<td>10th item</td>
<td>0</td>
<td>0.0</td>
<td>27</td>
</tr>
<tr>
<td>18th item</td>
<td>33</td>
<td>68.8</td>
<td>12</td>
</tr>
</tbody>
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**Table 1: Results of the analyses of three items in the questionnaire**

The answer of a participant \((P_{18})\) for the 18th item was categorized as correct since it presented the difference of the two concepts effectively.

\[
P_{18}: \text{Equation is an algebraic expression which holds for particular real number(s) while (algebraic) identity is an algebraic expression which holds for all real numbers.}
\]

The answer of another participant \((P_{31})\) for the 18th item was categorized as partially correct since the teacher considered all equations as if they were just first degree equations.

\[
P_{31}: \text{Equations hold for just one value (i.e., If } 3x + 5 = 8, \text{ then } x = 1). \text{ However, identity holds for all values of the unknown.}
\]
The results of the analyses for the 6th item (see Figure 2) are also presented in Table 1. The results revealed that the teachers were incompetent in some of the areas such as finding and correctly expressing the solution set of equations and identification and correction of students’ wrong ideas and misconceptions.

![Image of the 6th item in the questionnaire]

Figure 2: The 6th item in the questionnaire

The answer provided by one of the participants (P₃₄) was categorized as correct since the teacher stated that the solution set could not be real numbers and gave a suggestion on how to show it to the students.

P₃₄: The solution set cannot be real numbers since the equations do not hold for each \((x, y)\) when \(x\) and \(y\) are real numbers. Therefore, it could be shown to students by substituting some \(x\) and \(y\) values which are real numbers but do not satisfy the equations.

The answer of another participant (P₃₂) was categorized as partially correct since the teacher said that the equation could not be solved.

P₃₂: The solution is false. The solution set of the equation system should be in the form of \((a, b)\). It cannot be real numbers. Those are not two different equations. The first equation is double of the second equation. The two are the same equations. That is, it cannot be solved.

Some of the answers for the 6th item were categorized into incorrect category if the teachers stated that the solution was correct without any explanation or if they gave invalid/missing explanations.

In the 10th item, the teachers were asked "Why the direction of the inequality sign is changed when both sides of the inequality \(−x < 7\) are divided by a negative number?" (see Figure 1). There were no correct answers given by teachers since none of the teachers used algebraic expressions for the solution (See Table 1). Rather, the teachers mostly used particular values for \(x\) to show the change of the direction of sign when both sides were divided by a negative number. The answer of one of the participants (P₃₅) is categorized as particularly correct since the teacher used particular values rather than algebraic expressions.

P₃₅: I would give particular values. For example, if we multiply or divide both sides of \(2 < 5\) with \(-1\), the inequality will be \(−2 > −5\). By considering the values of the numbers, we can see that the inequality sign should be changed when both sides are multiplied or divided by \(-1\).

Also, the answer of one of the participants (P₄₄) is categorized as incorrect since the teacher stated that it was just a rule to be memorized. In addition, the invalid/missing answers were also categorized as incorrect.

P₄₄: I would say that it was a rule to be memorized. Thus, the inequality sign changes when both sides are divided by a negative number. Then, I would give examples with particular values.
Discussion and conclusion

Results of the study can be discussed under the two research questions. To begin with the first research question, results revealed that the middle school mathematics teachers participated in this study presented both difficulties and strengths in algebraic concepts. Most of the teachers performed success to make transition among different representations of algebraic expressions. Moreover, most of them were successful at items which require knowledge about particular algebraic concepts such as the difference between equation and identity. Although some studies concluded that the concept of equation and the concept of identity were frequently confused by pre-service middle school mathematics teachers (Altun, 2006; Güler, 2014), nearly all of the teachers differentiated both concepts accurately within the current study. Therefore, it might be inferred that teaching experience requiring to explain such conceptual differences to the students might get teachers competent not only in teaching but also in the knowledge of mathematical concepts. In some items, in-service pre-service teachers were more successful in comparison to the pre-service middle school mathematics teachers in the study of Güler (2014). This result might indicate that having teaching experience may get teachers to explain such mathematical expressions more conveniently by the help of their previous experiences. Those results might be suggestive for one of the dilemmas mentioned in the study of Doerr (2004) as the dilemma of experience. That is, the teaching experience might be more powerful than teacher education courses for the teachers' teaching of particular algebraic concepts. However, this result might be strengthened in such studies using further comprehensive instruments with more participant teachers.

According to the results for the second research question, analyses of the answers illustrated that some teachers had errors and misconceptions similar to those of students as it was also observed about in the study of Güler (2014). Teachers could not identify and explain the errors and misconceptions of students given in the scenarios in some of the items. Moreover, it was seen that teachers had some other misconceptions beyond the ones given in the scenarios. It was also observed that most of the teachers were struggling while explaining wrong answers of students and proposing an appropriate teaching method to correct their wrong answers. It may be inferred that one of the basic reasons of teachers’ inadequacy to propose an appropriate teaching method might be the lack of their knowledge of algebraic concepts. Since teachers could not provide a valid solution or explanation for the item, they could not provide a suggestion for the teaching of that concept (Güler, 2014). In some of the items, teachers were required to give conceptual explanations for the algebraic situations (i.e., “How would you explain to your students why $2^0$ is equal to 1?”). According to the results, nearly half of the teachers conceptually answered why $2^0$ is equal to 1. However, other teachers could not give a conceptual answer for that question or stated that "$2^0$ is equal to 1" is just a rule. Therefore, it might be inferred that some teachers might prefer to teach algebraic concepts by using rote learning consisting of rules to be memorized. To illustrate, to solve the 10th item, teachers were required to explain why the direction of the inequality sign changes its direction when both sides of the equation were divided by a negative number. Results indicated that there were nearly no conceptual answers given by the teachers for the 10th item. It was observed that some of the teachers explained it by saying that they were just rules which should be memorized like other mathematical rules, like the multiplication of two negative numbers' being equal to a positive number. Conversely, some teachers explained it by showing that it holds for many different values. However, none of them explained it by using algebraic procedures by
considering algebra as a mathematical generalization (Bednarz, Kieran, & Lee, 1996). This might be related with one of the dilemmas mentioned in the study of Doerr (2004). That is, teachers tend to choose the method, mostly the traditional method, while teaching mathematical concepts although they have learnt several methods for teaching mathematics.

Based on the results obtained in this study, it might be expressed as a common point that middle school mathematics teachers might have an incompetency in conceptual bases of some of the algebraic concepts (Ball, 1990; Güler, 2014; Tirosh, 2000). Teachers’ lack of conceptual knowledge about some algebraic concepts might lead to inability to conceptually explain these concepts to their students and recognize alternative ways of their students’ thinking regarding these concepts. Since teacher knowledge is considered as the first step to provide an effective teaching, it should be investigated and developed. As Cochran, DeRuiter, and King (1993) stated, teacher knowledge has a dynamic form and it develops continuously. Therefore, the results of the current study might be considered as a call for teacher education programs to strengthen aspects related to developing teachers’ conceptual knowledge of algebra and its teaching. On the other hand, as assessing in-service teachers’ knowledge for teaching (algebra) would require a more carefully constructed instrument and research design, the questionnaire itself and limiting the research design to survey type can be considered as a methodological limitation of this study. Even though the purpose would be to collect data from as much participant teachers as possible through an instrument, further studies should at least consider collecting data through interviews (with selected participants) requiring more elaboration on the answers given in the questionnaire and conducting classroom observations when the teachers are teaching the subjects/topics interested in the study. Furthermore, as much as it is challenging, more studies should focus on developing instruments for assessing teachers’ knowledge for teaching mathematics, particularly algebra.

References


