The notion and role of “detection tests” in the Danish upper secondary “maths counsellor” programme

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This paper presents and discusses a specific aspect of the Danish “maths counsellor” programme for upper secondary school, namely that of detection tests. More precisely, the purpose and design of a detection test is presented, as is the prospective counsellors’ use of the test. In the description, emphasis is placed on the ways in which detection tests assist in informing the maths counsellors in their work with students experiencing learning difficulties in mathematics.

Keywords: Detection test; “maths counsellor”; learning difficulties.

Introduction

The words “test” and “testing” are omnipresent in educational research and practice in general and in mathematics education in particular. However, the actual notions covered by these terms are very diverse, as are their roles and uses. Basically, the terms mean “critical examination of a person’s or a thing’s qualities”. There is an abundance of different purposes, goals and objects of testing, as well as a multitude of different approaches to and instruments for testing. Without attempting to outline a comprehensive theory of test and testing, one distinction is worth introducing in the context of mathematics education, a distinction between direct and indirect testing. In direct testing, the test object directly epitomises the very purpose and goal of the testing. If your purpose is to find out whether a given person can actually drive a car, a direct test consists of taking the person to trial in real car driving. In indirect testing, the test object is devised as an indirect indicator of – a probe into – something that is not identical to the test object itself, because this “something” is either inaccessible in direct terms or too large or too complex to be fully represented by the test object. So the test object becomes a proxy for – a representative of – the underlying, but necessarily indirect, object of testing. Mathematical learning, understanding, reasoning, modelling, and problem solving are just a few examples of such underlying objects of testing, for which a wide variety of test objects are only more or less well-chosen proxies. Mathematics education research and practice make extensive use of indirect testing, alongside direct testing.

The key issue concerning indirect testing is what relationships can be established between respondents’ responses to the test object and the real underlying object of testing. The generic question is: What do responses to the test object tell us about the respondent’s qualities in relation to the real underlying object of testing. Since, in indirect testing, these objects are not identical it requires a non-trivial amount of clarification of concepts, of interpretation and analysis, and oftentimes of independent empirical research to account for the inferences that can be justifiedly drawn from responses to the test object onto the underlying object of testing. The “detection tests” in focus in this paper are instruments for indirect testing. So, the above-mentioned generic question has to be specified for our context. Our research question then is: In what respects and to what...
extent does the detection test presented below allow for the detection of students with mathematics specific learning difficulties regarding mathematical concepts and concept formation?

Due to space limitations we are not able to fully answer this question here and to fully corroborate the answer. Instead, we confine ourselves to providing some key points in an answer, that is by describing the context in which the detection tests are used; by providing an overall description of what a detection test is; and finally by means of an illustrative and authentic example. It should be noted that although the actual content of the detection tests is based on research findings and issues considered in the mathematics education literature, the notion and role of “detection test” in the sense presented here have been introduced by us and thus have not been described previously.

The “maths counsellor” programme

This section is based on (Jankvist & Niss, 2016). The maths counsellor in-service teacher programme at Roskilde University (Jankvist & Niss, 2015) runs part time over three semesters (in total 30 ECTS – European Credit Transfer and Accumulation System), during which the upper secondary teachers – ideally – have a reduced teaching load at their schools. Each semester has an overarching theme: (1) concepts and concept formation in mathematics; (2) reasoning, proofs and proving; (3) models and modelling. These themes were chosen both because they are significant to upper secondary mathematics education in Denmark, as is spelled out in the national curriculum documents, and because they epitomise key aspects of the eight mathematical competencies in the Danish KOM-project (Niss & Højgaard, 2011), which constitutes the theoretical foundation of the maths counsellor programme. The teachers’ work in each semester is structured in terms of three different phases: (1) to identify (i.e., detect and select) students with genuine learning difficulties in mathematics; (2) to diagnose the learning difficulties of the student(s) identified; and finally (3) undertaking intervention according to the diagnosis arrived at with respect to the individual student.

At the very beginning of each semester, the teachers are equipped with a theme-specific detection test, consisting of questions and tasks for the students in relevant classes at their schools. As will be exemplified below, these tests are developed by us and are informed by research literature regarding the specific theme. The purpose of the test is to assist the teachers in detecting students with potential learning difficulties in mathematics. Usually, each teacher detects several such students, some of whom are selected for being offered maths counselling with the aim of rectifying or reducing the observed difficulties during the semester. This typically leads to the identification of 1-4 students per class in need of, and also interested in, receiving counselling. When speaking of mathematics specific learning difficulties, we rely on our previous definition given in (Jankvist & Niss, 2015, p. 260), i.e. “those seemingly unsurmountable obstacles and impediments – stumbling blocks – which some students encounter in their attempt to learn the subject. These stumbling blocks include, but are not limited to, a wide range of misconceptions, misinterpretations, misguided procedures, inadequate beliefs etc. with regard to established notions of mathematics. We do not include general learning disabilities, cognitive or affective disorders and the like.” The purpose of the counselling is not to motivate unmotivated students, but to assist those who work hard in mathematics on a daily basis but do not succeed.
In the diagnosing phase, the participating teachers – strongly assisted by the research literature they read as part of the programme (see Jankvist & Niss, 2015) – employ self-constructed tasks, interviews, etc., to come to grips with the nature and origin of the students’ mathematics specific learning difficulties. Taking the diagnosis as the point of departure and with support from the research literature and supervision by us, the teachers design and implement an intervention scheme for the students selected. The intervention scheme also includes steps which enable the counsellors to “measure” in what respects and to what extent the intervention has worked as anticipated for the selected students. For each semester, groups of 2-3 teachers write up a report. After the completion of the third semester, all three reports are combined into one, along with an introductory chapter. This final report forms the basis for a final oral exam at the university. The teachers who pass receive a diploma as certified maths counsellor.

What is a detection test – and what is it not?

A detection test, as designed for the Danish “maths counsellor” programme, is a set of maths questions to be answered by upper secondary student classes (grades 10-12) within a time frame of 60-90 minutes without time pressure. The questions are short, both in their formulation and in the sense that they neither require lengthy procedures or computations nor longwinded explanations. Moreover, the questions do not involve conceptually complex or technically involved mathematics beyond standard upper secondary school mathematics. However, the questions are usually not routine questions either. On the contrary, many of them are deliberately posed in such a way that they break the “didactical contract” of upper secondary mathematics and require students to think and act independently. Danish upper secondary school takes three years and students usually enter at the age of 16 after having completed ten years of mandatory comprehensive primary and lower secondary schooling. Upper secondary students can choose to have mathematics for one, two or three years; three years being the advanced level. Danish upper secondary school covers three streams: general, technical, and business.

The primary purpose of a detection test is to be one among several instruments for detecting students possessing genuine learning difficulties in mathematics, within the relevant theme of the programme. So, the focus is not primarily on detecting the difficulties themselves – even though the tests do have something to offer to that end as well, because the questions in a detection test are composed such that wrong answers, individually or in combination with others, may suggest the potential presence of particular kinds of learning difficulties with a student giving these responses. As mentioned, a detection test is not meant to stand alone. When it comes to detecting students with learning difficulties, other sources of information, e.g. the teacher’s prior knowledge of the students have to be taken into account as well. More precisely, a detection test may be seen as having three different roles. Firstly, in cases when the test, within a certain area or theme, points out students who by the teacher/counsellor were already suspected to have difficulties within that area, the role of the detection test is to strengthen the teacher’s observations. Secondly, in cases when the test singles out students who were not already detected by the teacher, the test serves to amplify and sharpen the teacher’s attention and to supplement his or her own observations of the students. Thirdly, it is also a purpose of the detection test to provide an initial support in pointing out the
specific sub-domains within the test’s theme, in which a detected student displays difficulties. Of course, students’ test responses may not only indicate difficulties within particular mathematical topics; students’ response patterns may also suggest overarching difficulties of a more principal or general nature. Thus, this third role of a detection test then typically is to provide inspiration for the following “diagnosis” (cf. later sections).

It is important to keep in mind that a detection test is not meant to be a fair test of the students’ attainment levels in the subject of mathematics, neither when it comes to content knowledge, skills, and proficiency, nor when it comes to mathematical competence at large or to inventiveness or special mathematical talent. Due to the fact that detection tests are designed with a different purpose in mind, several important aspects of the usual handling of mathematics – e.g. familiarity with concepts and facts, computational skills, or proficiency in solving standard routine tasks – are not in focus of the tests. Similarly, the test cannot be used as a screening test in the usual sense, attempting to chart students’ possession of various mathematical competencies. However, employed on a larger population of students, e.g. a year group in a given school, the test may of course be used as a screening test for the potential presence of mathematics specific learning difficulties pertaining the theme of the test, within this population, but the test is still much more focused than a general screening test for attainment level or competencies.

Even though the test contributes to singling out students with potential learning difficulties, it cannot determine, in itself, whether a given student actually possesses such difficulties. It is certainly possible to encounter poorly performing students whose erroneous answers are not due to mathematics specific difficulties, but to ill-will and shoddy job, lack of accept of the didactical contract with or in the test (e.g. because the test is not supposed to influence teachers’ marks, or because the questions are of a different nature than usually encountered by the students), a bad day on the time of testing, or maybe to much more general learning difficulties (or disabilities) that manifest themselves in several subjects, not only in mathematics. To determine whether a student detected by the test actually possesses mathematics specific difficulties, supplementary means must be applied as well, not least the teacher’s knowledge of the student.

Beside the fact that the test, for a student who has been “detected” by it, may provide important indications for a subsequent diagnosis of mathematics specific difficulties, the test is not a diagnostic test. It requires an independent diagnostic process to uncover the specific nature of observed learning difficulties as well as the sources actually responsible for them. Oftentimes, preliminary hypotheses concerning the nature of the difficulties, and what may have caused them, must be supplemented with – or even replaced by – other hypotheses as the diagnosis proceeds. This may be due to much more deeply rooted difficulties than the ones observed at first, e.g. regarding more fundamental mathematical conceptions and beliefs than those in focus of the detection test.

**An illustrative example of algebraic equations and equation solving**

As mentioned above, in each semester of the programme the maths counsellors are equipped with a detection test related to the theme of the semester. Hence, detection test 1 concerns mathematical concepts and concept formation (we intent to discuss detection tests 2 and 3 in subsequent publications). This test consists of some 57 questions (and sub-questions) on selected topics
relevant for Danish upper secondary school. These include: concepts of number (including fractions, decimals, negative numbers, irrational numbers); percent; algebraic expressions and transformations; equations (first and second degree, with different types of numbers as coefficients and solutions, and with the unknown on both sides of the equal sign); simple functions and aspects of the coordinate system; and finally a selection of mathematical conventions such as: different symbolic notations for fractions; the equal sign; the inequality sign; minus and negative numbers. Out of the 57 questions (with sub-questions) around ten questions concern equations and equation solving. In the following we shall focus on examples of this.

As suggested by various researchers (see e.g. Kieran, 2007), students’ difficulties in solving algebraic equations are of two rather distinct kinds. The first kind is related to transformation of equations – and algebraic expressions – by means of permissible operations, eventually leading to solutions. This not only involves knowing and understanding the scope and legality of the operations at issue, it is also to do with the nature and structure of the number domains implicated, the meaning of the equal sign, and the arithmetic operations involved, etc. The second kind of difficulty is to do with what an equation actually is, and what it means for an object to be a solution to an equation. Detection test 1 includes the following questions, among others: [17] Are there any values of \( a \) such that \( a^2 = 2a \)? [18] Are there any values of \( b \) such that \( 4b = 4 + b \)? [20] What is the solution(s) to the equation: \( 3x - x = 2x \)? [25] Is \( x = 0 \) a solution to the equation: \( 3x - x = 2x \)? [35a] Solve the equation: \( 3x + 20 = x + 64 \). [36a] Solve the equation: \( -6x = 24 \). For what \( x \) do we have \( 38x + 72 = 38x \)? Our purpose here is to illustrate two things: what the maths counsellor may learn from using the test on a larger population of students; and what the maths counsellor may learn about a single student from his or her answers to the test questions.

When a maths counsellor gives the detection test to a group of students, perhaps a larger cohort of students – say a class or a year group – certain patterns are likely to reveal themselves. For example questions 17 and 18 may tell us something about the students’ algebraic understanding, e.g. the students’ perception of how variables may and may not be denoted (anything other than \( x \) is often rejected as a variable). Questions 35a and 36a address the first kind of difficulty of solving algebraic equations, namely the operational aspect in relation to the number domains involved. Question 35a is an example of what Filloy and Rojano (1989) call a “non-arithmetical equation”, referring to the fact that the unknown appears on both sides of the equal sign. Question 36a may give rise to difficulties due to the appearance of the negative coefficient and division by a negative number, but also the situation of having to accept a negative number as a solution. On the one hand, questions 17, 20, and 25 may tell us about the second kind of difficulty mentioned above, i.e. knowing what a solution to an equation means, as well as about the consequences of the fact that an equation may have infinitely many solutions. On the other hand, they may also tell us something about the students’ conception of equality in relation to equations and equation solving. From extensive experience, we know that Danish students have difficulties with equations that have either no solutions or any number as a solution. Question 37 addresses another aspect of the second kind of difficulty. Despite the fact that the vast majority of students are not able to correctly answer question 20, a large number of students will say that 0 is indeed a solution to the same equation in question 25. More interesting, perhaps, are those students who are able to answer that all numbers satisfy the
equation $3x - x = 2x$, but still answer “no” to 0 being a solution. This may have to do with a belief that solutions are positive integers or be an aspect of more fundamental difficulties with 0.

To illustrate what an overview of a large student population may reveal, we provide table 1, which displays a binary (“correct-incorrect”) coding of 676 Danish upper secondary students’ responses from 2012 and 2013 (from all three levels and streams). For the 405 1st year students participating in the study we may confirm that questions 20 and 37 are indeed difficult ones, since 92.8% and 85.4%, respectively, cannot answer them correctly.

As an illustration of two maths counsellors’ use of the test in regard to equations and equation solving, we present the story of student Å (Christensen, 2016). Student Å followed the mathematics programme at intermediate level at a general upper secondary school. The two maths counsellors spotted student Å at the beginning of Year 1, and then worked with her for three consecutive semesters, while they themselves were enrolled in the maths counsellor programme. In relation to the above questions on equations, student Å answered incorrectly on both questions 17 and 18 (“no”), she left question 20 unanswered but answered question 25 incorrectly (“no”), and left questions 35, 36, and 37 unanswered. The two maths counsellors initially interpreted this as if she had difficulties with the transformation of algebraic equations and with algebraic expressions in general, since she also gave incorrect answers to: [6] What is $(a / b) \cdot (b / a)$? (Where neither $a$ nor $b$ is 0.) (Å: “$a^2 / b^2$”.) and [50] If $a = b$ is then $b = a$? (Å: “no.”).

Based on interviews which confirmed that student Å most certainly had difficulties in solving algebraic equations, and handling algebraic expressions in general, the two maths counsellors designed a series of small interventions focusing on solving various equations, arithmetic as well as algebraic ones, etc. (Filloy & Rojano, 1989). Soon, however, the maths counsellors began to suspect that Å’s difficulties had indeed deeper roots. Student Å found that negative numbers as well as fractions were “ugly”, and on one occasion she uttered that “0 that’s not a number!” Sometimes student Å had difficulty at distinguishing the operations of addition and multiplication. When trying to find the difference between two numbers, she counted on her fingers. When having to find how many times 8 divides 24, she answered “four” by counting “8, 12, 16, 24”, and even double checked the result by repeating the same count. Having to perform the division 24/6, she eventually gave up

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<td>1st year (405)</td>
<td>148</td>
<td>285</td>
<td>376</td>
<td>193</td>
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<td>149</td>
<td>346</td>
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<td>92.8</td>
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<td>36.8</td>
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<td>2nd year (196)</td>
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<td>166</td>
<td>172</td>
<td>87</td>
<td>54</td>
<td>44</td>
<td>139</td>
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<td>Error rate (%)</td>
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<td>84.7</td>
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<td>64</td>
<td>49</td>
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<td>18</td>
<td>43</td>
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<td>Error rate (%)</td>
<td>16.0</td>
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<td>33.3</td>
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Table 1: Binary coding of 676 student answers to selected questions from detection test 1 (coding by Morten Elkjær Hansen as part of his master’s thesis at Aarhus University, 2016)
and replied “I don’t know when the numbers are so big.” It turned out that student Å had fundamental difficulties with the concept of number, including understanding of numbers, number domains and handling of numbers. She appeared only to be on safe ground when operating with small natural numbers, where calculations can be performed on her fingers. Student Å’s difficulties extended to her (in)ability to correctly apply basic mathematical terms. Thus, on a later occasion she used the term “diameter” (of a pizza) as just another unit along with centimeter and millimeter.

Upon revealing the depth of student Å’s difficulties and acknowledging these to be the cause of her symptomatic difficulties with equation solving, the obvious question becomes whether the detection test might have provided us with some indications of this. In hindsight, even if there were several questions that Å left unanswered, the ones she answered erroneously do seem to corroborate her subsequently revealed learning difficulties: [7] Is the number – a positive or negative or is it not possible to decide this? (Å: “negative”). [13] Which number is larger: 13/3 or 13/4? (Å: “13/4”). [15a] What is x + 0? (Å: “0x”). [26] Round off 148.72 + 51.351 to an integer. (Å: “149 + 51 = 191”). [27] Which of the following fractions are equal: 1/4, 4/16, 4/12, 2/8? (Å: “4/12 and 2/8”). [52] If a < b is b > a? (Å: ”no”). [53] Is (a – 1) / (b + 1) = (a / b) – 1? (Å: ”yes”). 55. Is (a + 3) / (a + 4) = 3/4? (Å: ”yes”). 56. Is (a – 1) / (b – 1) = a / b? (Å: ”yes”). In total, the occurrence of the above erroneous answers, together with the responses to the previous questions on equations, indicates the presence of a manifest learning difficulty syndrome with student Å.

**The revelation of mathematics specific learning difficulty “syndromes”**

We now return to our research question, i.e. in what respect and to what extent do detection tests allow for the detection of students with mathematics specific learning difficulties – here exemplified by concepts and concept formation regarding equations and equation solving? As we saw in the case of student Å, she most certainly was detected to possess the potential learning difficulties as suggested by the test. Clearly, the counsellors’ first hypothesis concerning Å’s difficulties regarding concepts and concept formation was insufficient. However, we should keep in mind that this was the first time ever that these counsellors used the instrument of an indirect detection test. Once the maths counsellors become accustomed to the instrument and skilled in using it, they tell us that they are able to make much more accurate initial hypotheses – or even preliminary “diagnoses”. Indeed, having experienced a case like student Å, our two maths counsellors are able to make much more qualified initial hypotheses concerning students’ difficulties. Seeing the answers to the questions above on numbers, conventions, etc., these maths counsellors will no longer suspect a student “merely” to have difficulties with solving first degree equations; they will see this as a likely symptom of more deeply rooted and fundamental difficulties.

Indirect tests, such as the detection test outlined above, may mislead the interpreter of test outcomes in several ways. In the case of student Å we saw that the maths counsellors at first mistook the student’s apparent difficulties for her real, more fundamental difficulties. Another example, which we have also seen time and again, is where students are perfectly able to solve algebraic equations in an instrumental manner, but do not understand the relational aspects of the operations they perform or the very meaning of the solutions they arrive at (for references, see Jankvist & Niss, 2015). This is to say that if the aim is to “train monkeys” to find solutions to equations, then this is
certainly possible. Our aim with the indirect detection test is to go deeper, since “our” object of learning is more complex than to mechanically obtain a solution. Our aim is to pave the way for drawing conclusions that are much broader than what the test questions ask, taken at face value, e.g. we insert “spot probes” into aspects of students’ mastery of numbers and algebraic expressions and attempt to come up with hypotheses concerning their concept of number in general: if a student comes up with this and that erroneous answer, (s)he most likely possesses such and such learning difficulties; or if, on the contrary, the student can give correct answers on this particular set of questions, then it is fair to assume that (s)he has actually grasped, in a relational manner, something significant about the entities involved.

As illustrated above, an indirect test such as a detection test may function both on an individual student level and on larger populations. For example, in the case of student Å we noticed that she answered incorrectly to question 15 and question 50 (cf. above). This we interpret as an indication of student Å not believing 0 to be a number and possibly possessing misconceptions of equality. But how special are these misconceptions for a 1st year student like Å? The coding among the 405 1st year students displayed in table 1 revealed an error rate of 18.3 for question 15 and 13.6 for question 50. In addition, question 14, asking what $0 \cdot x$ is, which student Å answered correctly, has an error rate of 18.8 among the 405 1st year students. This is to say that if questions 14 and 15 are taken as markers of difficulties with the number 0, and if the population of the 405 students is representative, then it might be expected that more than one sixth of the students in a class at the beginning of 1st year will have the number 0 as a “stumbling block” in some sense. The indirect detection test may suggest the presence of syndromes, on an individual level as well as on the level of populations.

References


