On the edges of flow: Student engagement in problem solving

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Engagement in mathematical problem solving is an aspect of problem solving that is often overlooked in our efforts to improve students' problem solving abilities. In this paper I look at this construct through the lens of Csíkszentmihályi's theory of flow. Studying the problem solving habits of students within a problem solving environment designed to induce flow, I look specifically at student behavior when faced with an imbalance between their problem solving skills and the challenge of the task at hand. Results indicate that most students have perseverance in the face of challenge and tolerance in the face of the mundane, and use these as buffers while autonomously correcting the imbalance.

Flow

In the early 1970's Mihály Csíkszentmihályi became interested in studying, what he referred to as, the optimal experience (1990),

> "a state in which people are so involved in an activity that nothing else seems to matter; the experience is so enjoyable that people will continue to do it even at great cost, for the sheer sake of doing it." (Csíkszentmihályi, 1990, p.4)

In his pursuit to understand the optimal experience, Csíkszentmihályi (1990) studied this phenomenon among musicians, artists, mathematicians, scientists, and athletes. Out of this research emerged a set of nine characteristics common to every such experience (Csíkszentmihályi, 1990) – the first three of which are characteristics external to the doer, existing in the environment of the activity, and crucial to occasioning of the optimal experience.

1. There are clear goals every step of the way.
2. There is immediate feedback to one's actions.
3. There is a balance between challenges and skills.

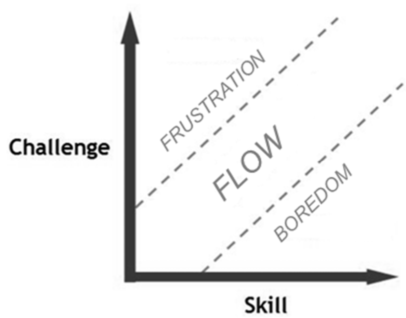

Fig. 1 Graphical representation of the balance between challenge and skill

The last of these – balance between challenge and skills – is central to Csíkszentmihályi's (1990) analysis of the optimal experience and comes into sharp focus when we consider the consequences of having an imbalance in this system. Csíkszentmihályi found that if the challenge of the activity far exceeds a person's ability they are likely to experience a feeling of frustration. Conversely, if their ability far exceeds the challenge offered by the activity they are apt to become bored. When there is a balance in this system a state of, what Csíkszentmihályi refers to as, *flow* is created (see fig. 1).

Flow is a powerful ways for us, as mathematics education researchers, to talk productively about the phenomenon of engagement in general, and the three aforementioned elements of flow gives us a way to think about the potential environments that occasion engagement in our classrooms in particular.

Williams (2001) used Csíkszentmihályi's idea of flow and applied it to a specific instance of problem solving that she refers to as discovered complexity. Discovered complexity is a state that occurs when a problem solver, or a group of problem solvers, encounter complexities that were not evident at the onset of the task, is within their zone of proximal development (Vygotsky, 1978), and occurs when the solver(s) "spontaneously formulate a question (intellectual challenge) that is resolved as they work with unfamiliar mathematical ideas" (p. 378). Such an encounter will capture, and hold, the engagement of the problem solver(s) in a way that satisfies the conditions of flow. What Williams' frame-work describes is the deep engagement that is sometimes observed in students working on a problem solving task during a single problem solving session.

Extending this work, I argued that engagement was an affective experience and used the notion of flow to look at situations of engagement extended over several days or weeks wherein students return to the same task, again and again, until a problem was solved (Liljedahl, 2006). The results of this work showed that although flow was present in each of the discrete problem solving encounters, what allowed the engagement to sustain itself across multiple encounters was a series of discovered complexities in each session linking together to form what I referred to as a *chain of discovery*.

More recently, I looked at the practices of two teachers through the lens of flow in general and their ability to set clear goals, provide instant feedback, and maintain a balance between challenge and skill in particular (Liljedahl, 2016a). From this a number of conclusions emerged. First, thinking about flow as existing in that balance between skill and challenge, as represented in figure 1, obfuscates the fact that this is not a static relationship. Flow is, in fact, a dynamic process. As students engage in an activity their skills will, invariably, improve. In order for these students to stay in flow the challenge of the task must similarly increase (see fig. 2).

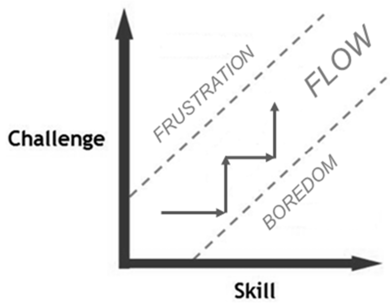

Fig. 2 Balance as a dynamic process

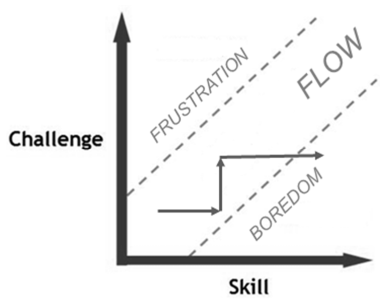

Fig. 3 Too fast an increase in skill

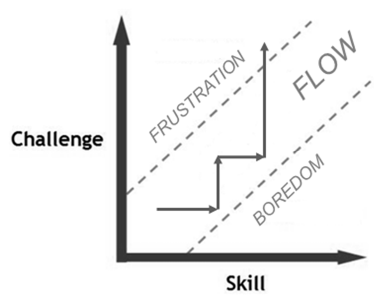

Fig. 4 Too great an increase in challenge

In a mathematics classroom, these timely increases of challenge often fall to the teacher. But this is not without obstacles. For example, if a student's skill increases either too quickly or too covertly for the teacher to notice that student may slip into a state of boredom (see fig. 3). Likewise, when the teacher does increase the challenge, if that increase is too great the student may become frustrated (see fig. 4). How teachers manage these situations of boredom and frustration is important. In Liljedahl (2016a) one of the teachers managed such situations synchronously, either giving hints or extensions to the class as a whole, usually after three groups finished or she got three of the same questions respectively. For most groups the timing of these hints and extensions was off, and not helpful in maintaining flow. The second teacher, however, managed these situations asynchronously, dealing with groups individually as they got stuck or completed a problem. Student engagement in the second teacher's class was visibly higher as he was maintaining flow through the constant and timely maintenance of the balance between ability and complexity for each group.

What I did not learn from this aforementioned research is how students cope with imbalance when the teacher does not provide help or extensions in a timely fashion. In the research reported here I look closely at exactly this phenomenon in general, and student autonomous actions and reactions in such moments of imbalance in particular.

Methodology

To get at this behavior I chose to observe students in a problem solving settings where student work was easily visible. To this end I strategically selected two senior high school classrooms belonging to two different teachers (Cameron and Charmaine), both of whom conducted their classrooms according to a teaching framework designed to shape their classroom into a space "that is not only conducive to thinking but also occasions thinking, a space that is inhabited by thinking individuals as well as individuals thinking collectively, learning together and constructing knowledge and understanding through activity and discussion" (Liljedahl, 2016b, p.364).

My earlier empirical work (Liljedahl, 2016b) on the design of such classrooms had emerged a collection of nine elements that offer a prescriptive framework to help teachers build such spaces. For the research presented here, five of these elements are particularly salient:

1. At the beginning of every class, students are assigned to a visibly random group (Liljedahl, 2016b, 2014) of two to four students.
2. These groups work collaboratively to solve a number of problems (usually) right from the beginning of the lesson.
3. This work is done with groups working at vertical non-permanent surfaces such as whiteboards, blackboards, or windows (Liljedahl, 2016b).
4. Students' flow is occasioned and maintained through the teacher's judicious and timely use of hints and extensions (Liljedahl, 2016a, 2016b).
5. At some point within this sequence of tasks the teacher brings the students together to debrief what they have been doing – either by going over one or more of the students' solutions or working through a new problem together with the class as a whole. This is timed so that every group is able to participate in discussion and benefit from the reification.

Taken together, both of these classrooms offered the affordances for me to easily observe students working within and an environment designed to occasion flow. The teachers were both managing engagement through the timely use of hints and extensions to maintain a balance between the challenge of an activity and the ability of each group. The student work was visible and there was enough autonomy afforded in the room that the students could take some kind of action when they found themselves in a situation where challenge and ability may be out of balance.

Data for this research were collected in Cameron's grade 12 Pre-calculus class and Charmaine's grade 11 pre-calculus class. Each class was visited five times over a seven week period in the middle of the second semester.

The data

Because the collection of video data creates such a narrow field of view, I instead used a variant of noticing (van Es, 2011) to scan the classrooms. Csíkszentmihályi (1990) characterizes flow as enjoyment, fluidity, and focus. These characteristics manifest themselves in the physicality of individuals and groups in flow and allows for the easy identification of flow and the absence of flow in a classroom. As per my research question, what I was looking for, then, were moments where an individual or a group was out of flow and where that individual or group was left to cope with this on their own. Once such a moment was identified I would focus in on that individual, or that group, taking detailed field notes and occasional photographs. When these moment seemed to wane I would conduct short, in-the-moment, interviews.

Csíkszentmihályi's theory of flow (1990) predicts that lack of flow is the result of a group of students' abilities exceeding the challenge of the task (see fig. 3) or the challenge of the task exceeding the abilities of a group (see fig. 4), resulting in the groups quitting, respectively, out of boredom or frustration. As such, flow served as the initial framework for analyzing the data. As it turns out, the theory was far from adequate for explaining all of the students' actions and reaction in the data. As such, I also used analytic deduction (Patton, 2002) to look more closely at students' actions and to group these actions into themes.

Results and analysis

From this analysis a series of six nuanced themes emerged, each marked by a different type of student action or reaction to being out of flow. In what follows I present cases exemplifying each of these themes as well as some general comments about similar cases.

When skills exceed challenge: The case of quitting

As mentioned, Csíkszentmihályi (1990) found that if a person's ability exceeded the challenge they are apt to become bored, and then quit out of this boredom. I found evidence of such behavior in Cameron's and Charmaine's classrooms.

Researcher	I notice you are not working on the assigned questions. What's up?
Mikaela	We did some of them.
Researcher	I saw that. I noticed that you did two very quickly. Took a little break from the math and then went back and did another one. I was sort of waiting to see if you would get back to it.
Allison	This stuff is easy. I'll finish it at home on my own.
Mikaela	It's actually too easy. I don't even think I will bother finishing it at home.
Allison	… Yeah. I probably won't either.

During the ten lessons I observed in Cameron's and Charmaine's classes I only managed to capture three other instances that I would say fall into the same category – quitting because the students were bored by seemingly too easy a collection of tasks.

When skills exceed challenge: The case of seeking increased challenge

Quitting out of boredom was not the only reaction to a situation where the skills of a group or of an individual exceeded the challenge of the task at hand. Some students opted, instead, to autonomously seek increased challenge. To exemplify this I look at a case from Cameron's class captured while students were working at the whiteboards in randomly assigned groups. During this part of the lesson Cameron moved around the room helping groups that were stuck (or had made a mistake) and giving more challenging questions to groups that were done. Before a group would get his help or the next question, however, he engaged the group in conversation to assess where the group's thinking was. This took time and sometimes groups that were done were left waiting.

Researcher	So, I notice that you guys are now on question 5 and your teacher has not visited you once. How are you getting your questions?
Ameer	We just look around and see what the next question is and do that one.
Researcher	What would your teacher say about that?
Carl	Um … he'd probably want to check to see that we got the previous one before giving us the next one …
Ameer	… but we are doing that.
Researcher	Why don't you just wait for your teacher to get here and give you the next question?
Carl	We're on a roll. And sometimes we have to wait a long time.
Researcher	Do you realize that you are doing the problems out of sequence from the order your teacher is giving them?
Colton	Oh really? That’s probably why some were so hard.

This was a very common reaction in both Cameron's and Charmaine's classrooms. Rather than wait for their teacher to give them the next questions groups were opting, instead, to move forward on their own by pulling the next question from groups that were ahead of them. This was facilitated by the visible nature of the work on the vertical surfaces.

When skills exceed challenge: The case of tolerance in the face of the mundane

An altogether different reaction to being tasked with doing easy and redundant questions is to just do them – without quitting and without seeking to increase the challenge. I observed such behavior in the case of Jennifer, who always worked at her desk on her own at the end of Charmaine's lessons.

Researcher	I have been watching you while I have been here. I notice that you always do a lot of questions. Can you tell me about that?
Jennifer	Yeah. I like to do a lot of questions. It's good practice. It's how I learn.
Researcher	So, are you looking for harder and harder questions to challenge yourself?

Jennifer	Not really. I just do all of them. So, if the teacher asks us to do 4a, I will also do 4bc and d and so on.
Researcher	Do you find them easy?
Jennifer	Yeah.
Researcher	How many do you do?
Jennifer	I just work the whole time at the end of class and then for maybe an hour at home.

I came to call Jennifer's behavior *tolerance for the mundane.* In my time in Cameron's and Charmaine's classes I saw two other girls who I suspect were very much like Jennifer in their approach to learning and their tolerance for the mundane. These girls also worked alone in their desks in the last part of every lesson.

When challenge exceeds skills: The case of quitting

Csíkszentmihályi's framework (1990) predicts that sometimes students quit out of frustration. I found three cases of this in Cameron's classroom – all near the beginning of class.

Researcher	I have been watching your group for a bit and I notice that you aren't working?
Robert	We gave up. This question is stupid.
Katrina	We tried, but we weren't getting anywhere. So we gave up.
Researcher	What do you think the problem is?
Shannon	This question is too hard.
Robert	… too hard. We don't get it.
Katrina	And the teacher hasn't come over to help us.
Researcher	What kind of help are you looking for?
Shannon	You know, a hint or something.
Researcher	What would a hint do for you?
Shannon	Help us understand the question.
Katrina	… or remind us a little bit about how to do it.

For this group the question they have been asked to solve exceeded their abilities and without any help from the teacher they gave up. Interestingly, the help they were seeking was not only to reduce the complexity of the task (understand the question), but also to increase their ability (remind us of what we have done in the past). In the ten lessons I observed, I only managed to capture four instances of a group giving up out of frustration.

When challenge exceeds skills: The case of seeking help

A much more common reaction to facing too great a challenge was for students to seek help. What this looked like, however, was much more subtle than simply asking the teacher for help.

Researcher	I notice that you have been moving about the room a bit. Why?
Michael	Oh. We were just stuck so we went over there to get some ideas.
Researcher	Did it help?

Michael	Oh yeah. We got it now.

This sort of behavior was endemic in both classrooms with too many occurrences for me to track. The vertical and visible work spaces facilitated the ability for groups to check their answers and get ideas. The random groups created the porosity (Liljedahl, 2014) that made the more active interactions and movement of ideas possible.

When challenge exceeds skills: The case perseverance in the face of challenge

But not all groups sought help when they were stuck.

Researcher	Question #5 was a tough one, huh?
Oliver	Yeah, that one took us a while.
Connor	In the end it wasn't that hard though. We were just missing something.
Researcher	Oh really. How did you figure it out?
Connor	We just kept at it and then we saw it.
Researcher	I noticed that your teacher came over to help. Did she help you?
Oliver	No, we wouldn't let her. We knew how to do it and we wanted to figure it out ourselves.

In all the lessons I observed I captured four instances where a group or an individual opted to not seek help, either from the teacher or the groups around them. I called this behavior *perseverance in the face of challenge*.

Discussion

The aforementioned six nuanced student reactions to being out of flow show that for different individuals and different groups the transitions from flow to boredom or frustration has variable immediacy. Some groups became bored or frustrated and quit. For these groups, Csíkszentmihályi's (1990) original representation of flow holds (see fig. 1).

For others, this transition was not as abrupt. Jennifer showed a great tolerance for the mundane as she spent long periods of time within a space where her ability far exceeded the challenge posed by the tasks she was working on. Likewise, Connor and Oliver demonstrating great perseverance while working on a task that presented too great a challenge for her ability. Taken together, these two cases, and the cases like them, indicate that for some students the boundary between flow and boredom and frustration is not as thin as Csíkszentmihályi's (1990) theory of flow would imply and is buffered by tolerance and perseverance (see fig. 5).

Other students used this buffer to avoid frustration or boredom as they sought to correct the imbalance between skill and challenge that they were experiencing. Carl, Ameer, and Colton used the groups around them to check their own answers and to seek out more challenging tasks when they were done. Similarly, Michael's group used the groups around them to access help when they were stuck. These groups, and the groups and individuals like them, managed to autonomously maintain the balance between challenge and ability. When their ability was too great they autonomously sought to increase the challenge (see fig. 6) and when the challenge was becoming too great they autonomously sought to increase their ability or decrease the challenge (see fig. 7).

The highly visible and collaborative environments created by the use of vertical non-permanent surfaces and visibly random groups were shown by the data to be instrumental in facilitating these autonomous actions.

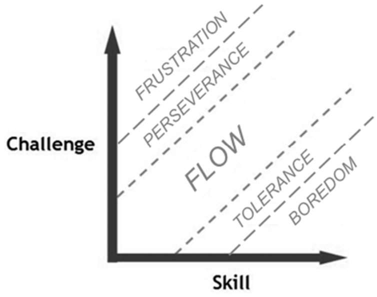

Fig. 5 Modified representation of flow

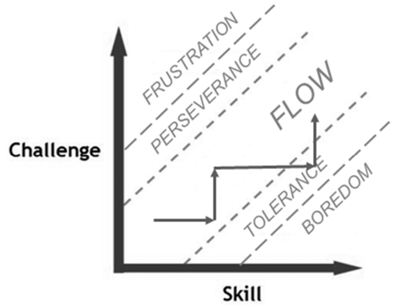

Fig. 6 Reaction to too great an ability

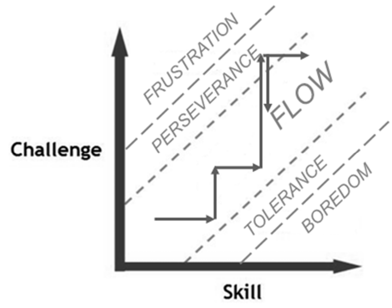

Fig. 7 Reaction to too great a challenge

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