Design research with history in mathematics education

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This paper describes a research project analysing design research projects with history of mathematics. As a background, the theory of design research is invoked. For the purpose of this paper, preliminary analyses of three publications have been made. In later phases, interviews will supplement text analyses to enable a discussion on both explicit and implicit considerations involved when designing materials with history of mathematics in mathematics education.

Keywords: Design research, history of mathematics, mathematics education.

Introduction

The study of how history of mathematics (HM) can contribute to mathematics education has been ongoing for a long time. From time to time, major efforts have been made to design materials for teaching mathematics with history of mathematics. Parallel to this, design research has emerged as an area of study in its own right. The purpose of this research project is to use the insights that recent literature of design research provides to study how design have been done with HM.

Design research

In a recent ICMI Study on task design in mathematics education, Kieran, Doorman and Ohtani (2015) outlines the history of "design-related work" in mathematics education. Design efforts have had many forms and names, but I will take as my starting point Malcolm Swan's encyclopaedia article on design research (Swan, 2014). He defines design research in this way:

> [Design research] is a formative approach to research, in which a product or process (or "tool") is envisaged, designed, developed, and refined through cycles of enactment, observation, analysis, and redesign, with systematic feedback from end users. In education, such tools might, for example, include innovative teaching methods, materials, professional development programs, and/or assessment tasks. Educational theory is used to inform the design and refinement of the tools and is itself refined during the research process. Its goals are to create innovative tools for others to use, describe and explain how these tools function, account for the range of implementations that occur, and develop principles and theories that may guide future designs. Ultimately, the goal is *transformative*; we seek to create new teaching and learning possibilities and study their impact on teachers, children, and other end users. (Swan, 2014, p. 148)

I choose to lean on Swan's definition and use the phrase "design research" here. Others use "task design" for similar efforts – task design is not to be understood as merely designing tasks:

> […] designing a task or task sequence in isolation from consideration of the design of the instructional culture in which the task is to be integrated may be of quite limited value – somewhat analogous to expecting a bird to fly with just one wing. (Kieran et al., 2015, p. 61)

Based on a selection from the literature on design research (including task design), I will focus on four dimensions: the goal; theories, values and design principles; testing; and end result.

First, what is the goal of the project? As seen in the quote above, the goal is to improve something (a product, process or tool) – for instance it could be to create materials (based on history of mathematics) that will improve how geometry is taught.

Design can be seen as an art or as a science (Kieran et al., 2015, p. 62) or as both. Seen as an art, creativity is an important factor, seen as a science, design will be based on previous theories. In addition, values will always play a role: "the frames and principles used in task design are intimately related to aims of mathematics education" (Kieran et al., 2015, p. 65). The role of theories is debatable, for instance Burkhardt (2013) stresses that "strong theories" are often overestimated in education, and argues for "phenomenological theories for specific areas" (p. 233). Kieran et al. (2015) distinguish between three levels of theoretical frames: Grand Theoretical Frames (such as the constructivist), Intermediate-Level Frames (such as the Theory of Didactical Situations) and Domain-Specific Frames (such as theoretical frames concerning specific parts of mathematics). Based on such theories, as well as on values, design principles are often developed for the research project. Thus, the second dimension is which theories, values and design principles are involved.

Kieran et al. (2015) also makes a further distinction: "*Design as implementation* focuses attention on the process by which a designed sequence is integrated into the classroom environment and subsequently is progressively refined, whereas *design as intention* addresses specifically the initial formulation of the design" (p. 28). In design as implementation, the testing in "cycles of enactment, observation and redesign" (Swan, 2014, p. 148) has a key role. Moreover, Burkhardt (2013) states that there is a "crucial difference" between exploration of teaching possibilities by a researcher and testing "what can be achieved in practice by typical teachers with available levels of support" (p. 207). He claims that impact on practice at least requires involving "typical teachers" in testing. The third dimension is therefore what the role of testing is in the project, who is doing it, in what way and in how many cycles.

The fourth dimension concerns the results of the project. The result may be the designed product that can be used by others. Often, local theories about the designed material are also developed:

> Potential users of a curriculum should know what conditions are necessary for its successful implementation, so they can make sure the conditions are in place […]. It is the development team's job to discover and provide this information in the later stages of development and from use in the field. (Burkhardt & Schoenfeld, 2003, pp. 6–7)

In addition, researchers also want to contribute to more general theories of mathematics education: "Design experiments […] are about improving both theory and practice" (Schoenfeld, 2014, p. 404)

For several reasons, design research studies do not always conform to the definition. For financial reasons, testing cycles are often reduced to a minimum. There are also political factors; theories seem to be valued more than practical solutions. When publishing or applying for grants, theoretical results may be stressed more than design results. However, this may be changing, as signalled by the introduction of the ICMI Emma Castelnuovo Award – an award "for excellence in the practice of mathematics education". Burkhardt (2013) contrasts the situation in education with medicine, where the development of new medicines and treatments are valued as much as new theories.

Kieran et al (2015) concluded with a note that "knowledge about design grows in the community as design principles are explicitly described, discussed, and refined" (p. 73–74). This is exactly my motivation for looking at how design is conducted when history of mathematics is concerned.

History of mathematics in mathematics education

Jankvist (2009) shows that the literature on history of mathematics in mathematics education was for a long time dominated by "publications advocating […] for history in mathematics education" and "publications describing either concrete uses by teachers or developments of teaching materials" (p. 13). Some publications in the latter category can be seen as small design research studies, but were mostly based solely on reflections by the teacher-researcher. By adding systematic testing and data collection, the projects can become empirical studies on the "effectiveness" of history of mathematics. Recently, there have been a number of these, and they are often focusing on generating theory rather than the development/design of material (although design nonetheless plays an important part). Alternatively, putting more weight on the development part, they can become fully-fledged design research studies. There does exist a small number of large-scale design research projects, for instance the Historical Modules project (Katz & Michalowicz, 2005).

Research questions

The main research question of this study is: What are characteristics of the design projects that include history of mathematics?

The design research perspectives are used to analyse the projects to shed light on what is considered important by researchers and the community. I will base the analyses on the four dimensions discussed above: the goal; theories, values and design principles; testing; and result.

As not all these dimensions are likely to be described explicitly in written articles about the projects, there are two sub questions: a) How is this process presented in writing? b) What considerations are involved which are not explicitly included in the written results?

In addition, this project may give suggestions on ways in which the literature on design research can contribute to HPM design projects and vice versa.

Methods

The project has three phases. In the first phase (reported in this article), I analyse three publications describing efforts in designing materials for teaching mathematics with history. The analysis is twofold, the texts are analysed in accordance with the categories of the design research literature, and also to find additional considerations not included in the design research literature that I have surveyed. The first phase can be regarded as a "pilot" to see if the approach seems worthwhile. In the second phase, a more thorough literature review is done and more texts are included, whereupon a more thorough analysis is done. In the third phase, interviews are conducted with researchers from selected design projects to identify considerations absent in the published texts.

For the first phase, three texts were chosen: Weng (2008), Barnett, Lodder, Pengelley, Pivkina, & Ranjan (2012) and Jankvist (2009). They were chosen because they are different in scope, target group and context, and could therefore be expected to provide diversity. Two of them are not design research studies on the face of it, thus the analysis can give me a clue as to whether including such other design-related studies in my analyses are worthwhile.

Preliminary results

Weng: Using history of mathematics in Singapore

Weng (2008) gives an overview of the use of history of mathematics in Singapore, while section 6 of the article describes "an action-research based case study" in which the author developed and gave a course using history of mathematics.

Goal: The stated goal of the study was "integrating history of mathematics into the teaching and learning of mathematics" and "investigate whether such a methodology help the students develop (or even enhance) a positive attitude" (p. 18). The article also includes a ten-page appendix giving examples from several projects, suggesting that the examples are assigned a value of their own.

Theory, design principles, values: The article refers to several potential effects of employing history of mathematics, but advocates the use of history of mathematics "to inculcate positive attitudes of the learner, as well as the teachers, towards mathematics" (p. 3). Weng proposes a "didactical framework", based on the thought that "the learner must make *intellectual leaps*" while mankind make "*historical leaps".* "[The] relationship between the mechanisms which are responsible for each of these leaps" (p. 13) is important. The intellectual leaps should be identified, "psychogenetical mechanisms" to help should be found, historical mechanisms associated to these should be identified, and historical points found which the historical mechanisms were employed to tackle. Identifying the historical points is called "sourcing", and concerns searching the literature and discussing with colleagues.

In the appendix, seven kinds of "implementation methods" are given – these could perhaps be seen as seven sets of design principles. The seven are historical snippets; primary sources; worksheets; historical packages and enrichment programmes; experimental activities using ancient instruments and artefact; outdoor experiences; integration into modes of assessment.

Testing: The course was taught (once) by the researcher himself. The article includes results from students' and teacher's logs and a student survey.

Results: The stated result of the case study was that the historical approach was effective concerning belief and perseverance. However, as mentioned earlier, some of the materials created are given as examples in the appendix, and there are also examples of "evaluations" connected to the concrete examples: "[…] students appeared motivated since this approach *replaced* the usual, *re-orientated* their mathematical perspective and promoted *cultural understanding*." (p. 35)

There is no discussion of which contexts the examples given could be suitable in, but there is discussion on the Singaporean context, including data on teachers' attitudes and a lament on lack of teacher training in history of mathematics, lack of curriculum time and lack of assessment rubrics. This could perhaps be helpful for others to see whether their context is similar to the Singaporean.

Barnett et al.: Designing Student Projects via Primary Historical Sources

Goal: The project described in Barnett et al. (2012) builds upon an earlier design research project (a "pilot program") in which "over a dozen historical projects for student work in courses in discreet mathematics, graph theory, combinatorics, logic, and computer science" (p. 189) were developed. In the new project, "additional projects based on primary sources are being developed, tested, evaluated, revised and published" (p. 189). The goal was thus to develop these resources, with the aim "to recover motivation for studying particular core topics by teaching and learning these topics *directly* from a primary historical source of scientific significance" (p. 190). The article was written while the authors were in the second year of the four-year project.

Theory, design principles, values: The article does not give an overview of the theory it is building on, instead just stating that "Much has already been written about teaching with primary historical sources", and then referring to chapter 9 of the 10th ICMI Study. Some design principles are given:

> each historical project is built around primary source material which serves either as an introduction to a core topic in the curriculum, or as supplementary material to a textbook treatment of that topic. Through guided reading of the selected primary source material and by completing a sequence of activities based on these excerpts, students explore the science of the original discovery and develop their own understanding of the subject. Each project also provides a discussion of the historical exigency of the piece and a few biographical comments about the author to place the source in context. (p. 190)

In addition, fifteen "pedagogical goals guiding the development" are given. They include "students' verbal and deductive skills", "moving from verbal descriptions […] to precise mathematical formulations", "the organizing concept behind a procedure", "understanding of the present-day paradigm [and] standards", "attention to subtleties", "students' ability to equally participate", "offer diverse approaches", "provide a point of departure for students' work", "more authentic (versus routine) student proof efforts", "a human vision of science and of mathematics", "a framework for the subject", "a dynamical vision of the evolution of mathematics"; "greater understanding of its roots" and "engender cognitive dissonance (dépaysement)" (p. 190).

Testing: The testing is done "by faculty at twenty other institutions" (p. 189), but no more detail is given in the article on the procedure, number of iterations and so on.

Results: The projects are published online at http://www.cs.nmsu.edu/historical-projects/, including "notes to the instructor" and comments from users of the projects. The article includes some experiences from the implementations (p. 199–200), including some comments from students and some possible ways of using the materials. This approach to using history to teach mathematics "is effective in promoting students' understanding of the present-day paradigm of the subject" (p. 200).

Jankvist: Using History as a 'Goal' in Mathematics Education

Jankvist (2009) is a dissertation, and therefore has more room for (and demand of) a clear theoretical underpinning than the articles. Moreover, Jankvist's project is not design research as such – to the contrary, the project is an empirical research study whose stated goals are to answer general questions, with materials only as "a byproduct" (p. 8). The three research questions are

> RQ1. In what sense, to what extent, and on what conditions is it possible to have upper secondary students engage in meta-issue discussions and reflections of mathematics and its history in terms of ‘history as a goal’?
>
> RQ2. In what sense and on what levels may an anchoring of the meta-issue discussions and reflections in the taught and learned subject matter (in-issues) be reached and ‘ensured’ through a ‘modules approach’?
>
> RQ3. In what way may teaching modules focusing on the use of ‘history as a goal’ give rise to changes in students’ beliefs about (the discipline of) mathematics, or the development of new beliefs? (p. 45)

However, for two reasons it makes sense to regard this project as having a design research project at its core. Firstly, his way of answering his research questions is by designing and testing two modules. Secondly, the developed materials are interesting results in their own right, as evidenced by their being published in full (Jankvist, 2008a, 2008b). Thus, in this analysis, I will look at the design parts of Jankvist (2009) as an example of design research.

Goal: The goal of the design research part follows directly from RQ1–3; to design teaching modules engaging students in meta-issue discussions and reflections of mathematics and its history, anchored in in-issues, changing students’ beliefs about mathematics in the process.

Theory, design principles, values: The theories are treated systematically and in detail. First, he gives his categorization of the whys and hows. Then, he discusses Meta-Issues (inner and outer driving forces; pure and applied mathematics; epistemic objects and epistemic techniques; discovery versus invention; multiple developments), In-Issues (in particular Sfard’s theory of commognition) and Student Beliefs (stressing students’ beliefs about mathematics as a discipline and the role of reflection in changing beliefs). As Meta-Issues and In-Issues are part of what the students are supposed to reflect on, a thorough theoretical treatment of them is of particular relevance.

Design principles are not treated as systematically; they are found throughout the dissertation:

> Obviously, original sources have to be chosen with great care, depending on the educational level in question, in order to make sure that the students have a realistic chance of actually working with them. (p. 33)
>
> The historical cases chosen for [a modules approach] should […] be exemplary, e.g. in such a way that they embrace as many general topics and issues related to the history and historiography of mathematics as possible. (p. 89)
>
> [cases should be chosen] for which the in-issues could be built up in front of the eyes of the students in parallel with the explaining of the related meta-issues. (p. 94)

Other design principles are “using modern notation in the presentation of the mathematical in-issues” (p. 95), “setting the text of the teaching material with two different fonts; one for in-issues […] and one for meta-issues” (p. 95), and “Following their group discussions, the groups were to write essays on the topics in question and hand these in” (p. 95).

Jankvist also offers some of his “personal viewpoints”, such as that it is “important to provide students with a ‘picture’ of what mathematics in time and space is” and that “one must have some kind of understanding of the involved mathematics also” (p. 7).

Testing: The actual teaching was done by “a typical upper secondary mathematics teacher” (p. 96), being “coached” by the researcher (p. 115), but no teacher’s manual was written (p. 95). There was just one cycle, but the testing of the first module led to some changes in the second module. Most importantly, “[instead] of the introductory essay assignments, so-called historical exercises were introduced” (p. 157). Moreover, discussions with the teacher also led to at least one change, in that the researcher agreed to discuss the final essays with the class (p. 127).

An immense amount of data was collected: videos of the teaching and of focus group discussions, interviews with teachers and students, lots of hand-ins, including essays, and several questionnaires.

Results: The modules have been published, but not (as far as I know) in a new version informed by the results of the testing, although there are examples of details that were “ill-suited” (p. 150) and examples of new ideas; including role play (p. 202) or using the wording “on the shelves” (p. 203).

There is no attempt in the dissertation to describe conditions necessary for using the modules, except that “In other countries with different types of curricula, the possibilities for doing this may be somewhat limited” (p. 108). It is pointed out that although this “typical” teacher was coached by the researcher, she felt she lacked historical knowledge (p. 275). This makes it doubtful that other teachers with the same level of confidence would use the modules on their own – suggesting that having teachers collaborating with researchers to develop materials might be better (p. 304).

On the questions that the dissertation set out to answer, however, there are ample answers: Students were able to have discussions on meta-issues, anchored in in-issues. The essay assignments “appear to be a suitable setting for having the students engage in meta-issues” (p. 201). “[S]ome of the effects of choosing a newer history over an older one appear to be that it may be easier to relate to” (p. 281). Changes in students’ beliefs/views were observed.

Preliminary discussion

The three publications include the theoretical background to very different degrees – probably partly because of context and space restrictions. Therefore, I will not discuss this in detail here. Design principles, however, are detailed in all three publications. Some of these concern the parts of history to be chosen. In Weng’s case, specific “historical points” are found that will help students make “intellectual leaps”, while in Jankvist’s case, the historical cases should be “exemplary”, but without concern for whether the mathematics covered is already a central part of the curriculum. Barnett et al., on the other hand, does not discuss the choices of topics but seem to choose topics already central to the curriculum.

All three projects include testing to some degree, although they include different levels of detail. While Barnett et al. explicitly states that testing will be used to revise the materials, Jankvist gives examples of revisions that could be made but he does not make them. None of the publications give very detailed (testing-based) pointers on what “conditions are necessary for its successful implementation”, to quote Burkhardt and Schoenfeld. However, both Jankvist and Weng are concerned about the teachers' attitudes and knowledge, raising the question of whether the materials could be used by "average" teachers at all, without significant support.

For two of the publications, the materials produced are not presented as the main result of the studies. If this emerges as a pattern, it would be interesting to investigate whether this is because of the authors' opinions or because of external factors such as the expected format of research texts.

Conclusion of the first phase ("pilot")

The first phase of this project establishes that there are significant differences in the goals, theoretical underpinnings, design principles, testing and results in the three chosen texts. Bringing such differences into the foreground may contribute to a discussion which can, in turn, benefit future design research projects.

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