Inquiry-based teaching approach in mathematics by using the history of mathematics: a case study

Areti Panaoura

Frederick University, Cyprus; pre.pm@frederick.ac.cy

The use of the history of mathematics during an inquiry-based teaching approach is expected to multiply the positive effects on students' learning. The present work investigated a "typical" teacher's difficulties while trying to use the history of mathematics as a teaching tool during inquiry-based teaching activities. Two examples were used which presented at the textbook of the 5th grade of primary education in order to observe the teaching practices. Results indicated that the teacher had difficulties to understand how students could investigate a mathematical concept by integrating the history of mathematics and how the study of the history would enable them to construct the new-acquired knowledge. The respective knowledge of the domain seemed to be a prerequisite in order to be able to use the history of mathematics fluently and flexibly in a learning environment which asked students to explore or investigate the mathematical concepts.

Keywords: case-study, inquiry-based teaching approach, history of mathematics

Theoretical background

Mathematics education aimed to develop pupils' abilities to think logically, critically and creatively by recognizing that mathematics permeates the world around and by recognizing the power and the beauty of mathematics. We believe that a central key for those aims is the appreciation of the multicultural and the historical perspective of mathematics which faced the tendency to understand it as a formal science which has already been discovered. Using authentic problems from the history of mathematics provides experiences for students to actively engage in classroom discourse (Gulikers & Blom, 2001), and to realize the role of the construction of the science of mathematics.

Since 2009, in the context of the CERME we have the appearance of the specific group which discusses the role of using the history of mathematics, the theoretical framework, the teaching practices and the respective learning results. In a meta-analysis, Butuner (2015) included 56 researches in Turkey and abroad in order to reveal the influence of using the history of mathematics on success. By the same way numerous articles have been published in scientific journals and many conferences have been done, without exhausting the discussion on how to use in a more productive way the history of mathematics in order to fulfill the aims of mathematics education. Recently there was a special thematic issue of the Menon Journal of Educational Research about the use of the history of mathematics in mathematics education. The emphasis concentrated on the educators' experiences, beliefs and practices on using the study of the historical aspects of many different concepts for the teaching of mathematics in different ages. There are many studies on the

level of higher education (e.g. Weng-Kin, 2008) and on the level of secondary education (e.g. Kaygin, et al., 2011, Lim & Chapman, 2015) and fewer about the primary education.

The present work joined the use of the history of mathematics at a specific grade in primary education, with the aim of using the inquiry-based approach as a teaching method which was supposed to enable students explore and investigate the new mathematical concepts. At the Curriculum of Mathematics which was constructed in 2011 for the primary education in Cyprus, the use of the history of mathematics was suggested in order to develop students' positive beliefs about mathematics and the usual use of inquiry-based teaching was proposed as the main teaching approach. The two central concepts for the inquiry-based teaching approach which were proposed were "investigation" and "exploration". A case study of a "typical teacher" was used in order to investigate the two specific research aims: a) to examine his knowledge and beliefs on using the history of mathematics in an inquiry-based framework and b) to reveal the teaching practices which are used and the teaching difficulties which are faced during the implementation of the innovation.

The history of mathematics as a teaching tool

In 2000 the International Commission on Mathematics Instruction has set up a study on the role of the history of mathematics in the teaching and learning of mathematics. The main intention was to study the role of the history of mathematics in relation to the teaching and learning of mathematics to the teacher training. Jankvist (2009) explains the use of the history of mathematics both as a tool and as a goal and suggests that introducing the history of mathematics in school curricula enhances learners' motivation, promotes favoured attitudes, and draws attention to possible obstacles faced in the generation and evolution of mathematical concepts. As a pedagogical tool it can serve as a guide to understand the difficulties students may encounter as they learn a particular mathematical topic (Haverhal & Roscoe, 2010). History of mathematics enables teachers to present to their students how mathematical ideas develop and to guide them appreciate mathematics as a creative disciplinary activity. Schubring and colleagues (2000) also posit that programs based on the history of mathematics could increase self-confidence in working with mathematical tasks and develop learners' ability to apply mathematical methods. A journey through the history of mathematics can also enable learners to construct mathematical meanings and support new conceptions about mathematics by changing learners' existing beliefs and attitudes (Dubey & Singh, 2013).

Jahnke (2000) suggests three general ideas which are suited for describing the special effects of studying a source on the teaching of mathematics: (a) the notion of replacement according to which mathematics is seen as an intellectual activity, (b) the notion of reorientation according to which history reminds us that the mathematical concepts were invented and (c) the notion of cultural understanding. As Siu (1997) claims, using the history of mathematics in the classroom does not necessarily increase students' cognitive performance, but "it can make learning mathematics a meaningful and lively experience, so that learning will come easier and will go deep" (p. 8). As Panasuk and Horton (2013) underline the learning of mathematics can be facilitated by studying the cultural significance of mathematics and understanding that "in the earliest stages of invention,

many of the mathematical concepts were extremely difficult to define, understand and accept for even the most gifted mathematicians" (p.38).

Although the mathematics teachers in the study by Lit and Wong (2001) were very supportive in using history in their teaching, Siu (1997), in an invited talk given at the working conference of the 10th ICMI study on the role of mathematics in mathematics education, offered a list of thirteen reasons why a school teacher hesitates to make use of the history of mathematics in classroom teaching such as "I have no time for it in class", "Students don't like it", "There is a lack of teacher training on it", "Students do not have enough general knowledge on culture to appreciate it", etc.

The inquiry-based teaching approach

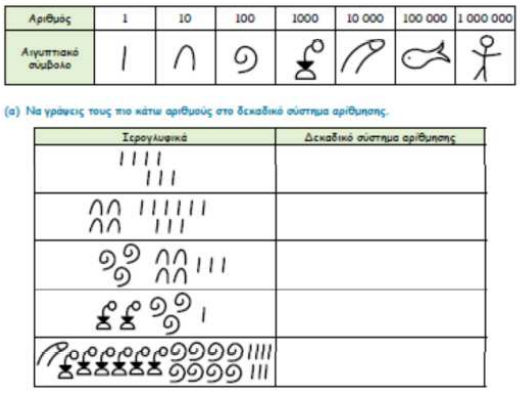
The inquiry-based approach in mathematics education is supposed to promote engagement and ownership and a "human view" of science as knowledge in the making (Savelsbergh et al., 2016). It requires teachers to use pedagogical methods which actively engage students in developing conceptual understanding of mathematical concepts (Chapman, 2011). The challenge for educational systems is to enable its teachers to adopt the values of the inquiry-based pedagogy. The scientific journal of *ZDM* in Mathematics Education has published a special issue in 2013 with nine papers focusing on inquiry-based mathematics education and their implementations, indicating that many questions remain unanswered.

Teachers need to develop their ability to foster student decision-making by balancing support and independence in thinking and working (NCTM, 2000). Classroom management is a crucial aspect of instructional quality (Taut & Rakoczy, 2016). Chin and Lin (2013) claim that there are obstacles and difficulties such as: (i) teachers did not experience inquiry-based learning in mathematics in their own school years, (ii) they do not have complete understanding of the inquiry-based teaching, (iii) there are practical constraints such as that the allocated teaching hours are not enough, (iv) the influence of teaching for success in tests.

Maab and Artique (2013) examine the implementation of the inquiry-based approach and look at its implementation through resources and professional development. They indicate that there is a need to promote a widespread uptake of inquiry-based approach in day to day teaching. One of the main emphases of the new proposed teaching model of Mathematics in the centralized educational system of Cyprus which is presented at the New Curriculum (NCM, 2011), is the use of "exploration" and "investigation" of mathematical ideas, as two dimensions of the inquiry-based teaching and learning approach. The whole idea is to introduce a mathematical concept by using an inquiry-based activity through which the teacher asks students to express their ideas and arguments, to communicate by using the language of mathematics. The emphasis is on using authentic and open-ended problem solving activities without only one correct answer and by respecting the value of inter-individuality.

Methodology

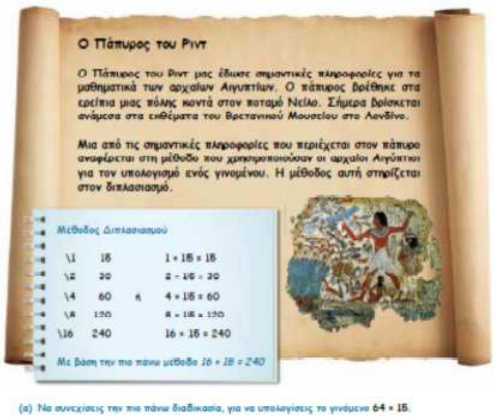
The emphasis of the present study was to examine the teaching practices used during the implementation of the inquiry-based activities by using the history of mathematics in authentic classroom situations. We chose to observe two lessons where the use of the history of mathematics was proposed by the textbook, at the 5th grade of primary education. We are referred to a centralized educational system where the Curriculum, the textbooks and the teaching materials are proposed by the Ministry of Education. A "typical" teacher was chosen after the first phase of the study which is not presented at the present paper. The criterion for the selection was his medium performance concerning his knowledge and beliefs about using the history of mathematics and the inquiry-based teaching approach in mathematics. He took part at a first phase of the project which collected data about teachers' knowledge and beliefs (details about the questionnaire are presented at Panaoura, 2016). We aimed to make the link between what he might say during an interview and what he actually did during the teaching. By using the case-study approach we emphasized the analysis of the teaching conditions in real-life classroom situations and the interpretation he proposed during a follow up interview. Firstly the teacher at the 5th grade was observed by the researcher and then semi-structured interviews were conducted in order to discuss the lessons. The lessons were chosen because an activity of using the history of mathematics for introducing a concept during an investigation was suggested by the school textbooks. The proposed activities are presented at the Figure 1.



Egyptians used the hieroglyphs in 3000BC which included 7 different symbols in order to represent the numbers. Write the numbers in the decimal numbering system.

A follow up task asks them to compare the two systems and write their comments

Unit 3, page 73



The Reed's papyrus gave us important information about the mathematics of the ancient Egyptians. One of them is the method of multiplication by using the doubling method. After studying the method, apply it in order to find out the result of 64X15 and then use the distributive property in order to find out the 13X15.

Unit 3, page 100

Figure 1: The activities as presented at the textbook (in Greek and in translation)

A protocol for the observation was constructed and used in order to concentrate the observer's attention on: a) teacher's guidelines at the introduction of the activity and his interventions while students were working and b) teachers' feedback on students' difficulties and mistakes. The semi-structured interviews with the teacher were concentrated on the practices he used and the difficulties he faced.

Results

The teacher's observation enabled us to concentrate our attention on the teaching practices he followed in order to use the inquiry-based approach during the teaching of numbers and operations, by using a historical perspective.

In the first case the teacher asked students to study the page, then they had to write few numbers by using the hieroglyphs and finally they were asked to transform other numbers into the decimal arithmetic system. After they presented a few numbers, their teacher asked them to discuss with the members of their group the similarities and differences of the two systems. The specific activity lasted for 10 minutes and then a whole class discussion was conducted. Teacher insisted by posing questions to guide them understand the limitations of the ancient Egyptians' numeric system. Many correct answers were given by the students and only one unexpected question was posed by a girl: "Today in Egypt people use these symbols or something which remind them the attempts of their progenitors?" The teacher explained why the ancient systems were not survived by repeating arguments which were presented previously by the students, such as the complexity of the symbols. Nevertheless he admitted that he was not able to answer whether there is something in Egypt today which is related with the specific system. He continued by showing his clock and the roman symbols on it, he explained that there were residues of arithmetic systems and symbols which were used in the past. He then asked students voluntarily to look in their free time for more information about the arithmetic system of the ancient Egyptians in order to be able to answer their classmate's question in three to four days. As he admitted during the interview there were some students who tried to find out more information about the numeric systems. They had not found anything about Egyptians; however they discover the Babylonians' impact on the way of measuring the time and the Latin numbers on buildings such as the German Parliament.

In the second case, it was the use of the ancient Egyptians' algorithm of multiplication. The teacher asked students to study individually the method which was presented and applied it at the multiplication 35X17. Few students were not able to continue after the 32X17. One of them continued by writing 3X17 and then she added the two products. Teacher said that it was a wrong solution because "Egyptians did not know how to find 3X17". The follow up dialogue is interesting:

Student: How is it possible to know 2X17, 4X17, 32X17 and they didn't know 3X17?

Teacher: They knew only to double the product.

Student: Why they did that?

Teacher: It was their algorithm.

Student: But the guideline at the book asked to use the distributive property to find the product. I had used it, 32X17 and 3X17.

Teacher: It is right today, but not for the ancient Egyptians.

Student: They were not clever.

It is obvious that the student did not understand that the method of the Egyptians depended on the property according to which when a factor is duplicated the whole product is duplicated and she was not able to understand why this method was easier for them rather than the algorithm which is used today. However it is important that she understood the use of the distributive property in mathematics. Actually this was the objective of the specific course and probably the teacher did not know that the history of mathematics was proposed in the specific case in order to enable students investigate and understand the use of the distributive property in multiplication. When the teacher was asked about the teaching aim and the respective learning aim he said:

Teacher: The history is used in order to understand that mathematics was created by humans.

Researcher: Yes, but they could understand this at the previous lessons, with the arithmetic systems.

Teacher: Here they can understand that complicated processes were used as well.

Researcher: Which was the impact of those processes on the development of mathematics?

Teacher: I don't know. However it is important for humans to study their past.

Researcher: Do you know which were the ancient Egyptians' occupations and where did they use mathematics?

Teacher: No, I am not sure, probably for their transactions.

The teacher used only naive teaching arguments for studying the history of mathematics without understanding that students by investigating the way the arithmetic properties were used, they could understand the use of those properties in order to simplify the used processes. He seemed to not have adequate knowledge about the cultural, political and economic framework of using the specific processes in order to be able to judge their utility.

Discussion

Teachers will continue to be expected to actively engage students in inquiry-based experiences. At the same time most of the Curriculum will continue to ask teachers to use the history of mathematics as a teaching tool in order to enable students to understand the continuity and the development of mathematics in respect to the cultural circumstances. The current study provided evidence that although probably a teacher may express positive beliefs about the importance of the history of mathematics for the introduction or the understanding of mathematical concepts, he or she may face serious difficulties in implementing an inquiry-based teaching approach. Teachers

needs experiences during their school life or even during their pre-service training in order to be convinced for the results of the inquiry-based learning and the positive results of exploring and investigating the mathematical concepts through a historical perspective.

The historical approach is supposed to encourage and enable students to regard mathematics as an intellectual process and an on-going activity of individuals (Grugnetti & Rogers, 2000). The prerequisite is to enable them to understand how mathematics thinking and applications developed in different cultures, in response to the needs and thinking of different societies. As it is obvious from the present qualitative study, there are fundamental problems in the implementation of this objective in relation to other main objectives such as the use of the inquiry-based teaching approach. In the case of the New Curriculum in the educational system of Cyprus the history of mathematics is proposed to be used as a tool in teaching the students topics or concepts within the curriculum (Jankvist & Kjeldsen, 2011).

The present study is just a part of a project which investigates the use of the inquiry–based approach. Much more research has to be developed in order to relate the teachers' knowledge and beliefs about the use of the history of mathematics with their beliefs and knowledge about the inquiry-based approach in different grades Teachers' knowledge and beliefs are the official targets of educational reform (Uwe, Espinoza & Barbe, 2013). Emphasis has to be given on studying further teachers' difficulties in implementing the inquiry-based teaching approach in general and in the case of using the history of mathematics in particular, by examining the results of intervention programs in real classroom actions, with an emphasis on facing the teachers' difficulties

References

Butuner, S. (2015). Impact of using history of mathematics on students' mathematics success: A meta-analysis study. *Mevlana International Journal of Education, 5(2),* 78-95.

Chapman, O. (2011). Elementary school teachers' growth in inquiry – based teaching of mathematics. *ZDM Mathematics Education, 43,* 951-963.

Chin, E. & Lin, F. (2013). A survey of the practice of a large – scale implementation of inquiry – based mathematics teaching: from Taiwan's perspective. *ZDM Mathematics Education, 45,* 919-923.

Grugnetti, L. & Rogers, L. (2000). Philosophical, multicultural and interdisciplinary issues. In J. Fauvel & J. van Maanen (Eds.) History in mathematics education: the ICMI study (pp. 39-62). Dordrecht: Kluwer.

Haverhals, N. & Roscoe, M. (2010). The history of mathematics as a pedagogical tool: Teaching the Integral of the secant via Mercator's projection. *The Montana Mathematics Enthusiast, 7(2),* 339-368.

Hegarty-Hazel, E. (1986). *Lab work SET: Research information for teachers, number one.* Canberra: Australian Council for Education Research.

Hwang, G. J., Wu, P. H., Zhuang, Y. Y., & Huang, Y. M. (2013). Effects of the inquiry-based mobile learning model on the cognitive load and learning achievement of students. *Interactive Learning Environments, 21*(4), 338-354.

Jankvist, U. T. (2009). A categorization of the "whys" and "hows" of using history in mathematics education. Educational Studies in Mathematics, *71*, 235-261.

Jankvist, U. T. & Kjeldsen, T. H. (2011). New avenues for history in mathematics education: mathematical competencies and anchoring. *Science & Education, 20,* 831-862.

Kaygin, B., Balcin, B., Yildiz, C. & Arslan, S. (2011). The effect of teaching the subject of Fibonacci numbers and golden ratio through the history of mathematics. *Procedia – Social and Behavioral Sciences, 15,* 961-965.

Lim, S. y. & Chapman, E. (2015). Effects of using history as a tool to teach mathematics on students' attitudes, anxiety, motivation and achievement in grade 11 classrooms. *Educational Studies in Mathematics, 90(2),* 188-212.

Maab, K. & Artigue, M. (2013). Implementation of inquiry-based learning in day-to-day teaching: a synthesis. *ZDM, 45(6),* 779-795.

National Curriculum in Mathematics (2011). Ministry of Education. Cyprus.

National Council of the Teachers of Mathematics (2000). *Principles and Standards for School Mathematics.* Reston:VA.

Panaoura, A. (2016). The history of Mathematics during an inquiry-based teaching approach. *Menon: online Journal of Educational Research, 5,* 130-145.

Panasuk, R. & Horton, L. (2013). Integrating history of mathematics into the classroom: Was Aristotle wrong? *Journal of Curriculum and Teaching, 2(2),* 37-46.

Savelsbergh, E., Prins, G., Rietbergen, C., Fechner, S., Vaessen, B., Draijer, J. & Bakker, A. (2016). Effects of innovative science and mathematics teaching on student attitudes and achievement: A meta-analytic study. *Educational Research Review, 19,* 158-172.

Schubring, G. Cousquer, E., Fung, C., El-Idrissi, A., Gispert, H., Heiede, T., et al. (2000). History of mathematics for trainee teachers. In J. Fauvel, & J. Maanen (Eds.), *History in mathematics education – The ICMI study* (pp. 91-142). Boston, MA: Kluwer.

Siu, M. K. (1997). The ABCD of using history of mathematics in the (undergraduate) classroom. *Bulletin of the Hong Kong Mathematical Society, 1*(1), 143–154.

Taut, S. & Rakoczy, K. (2016). Observing instructional quality in the context of school evaluation. *Learning and Instruction, 46,* 45-60.

Uwe, G., Lorena, E., & Joaquie, B. (2013). Being a mathematics teacher in times of reform. *ZDM Mathematics Education, 45,* 535-545.

Weng-Kin, H. (2008). Using history of mathematics in the teaching and learning of mathematics in Singapore. *Paper presented at the 1st Rafles International Conference on Education.* Singapore.