Positioning and emotions in learning Algebra: The case of middle-achieving students

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Abstract. *Mathematics education researchers utilize different theoretical frameworks to study the role of affect in learning mathematics. This paper utilizes a discursive framework (*Evans, Morgan, & Tsatsaroni, 2006*) to study middle-achieving students' emotions in learning, utilizing technology, the topic of 'the quadratic function as a product of two linear functions'. The students' learning was videoed and then transcribed to be analyzed according to the discursive framework. The research results indicate that members in the middle-achieving groups claimed the collaborator positioning in order to learn the mathematical topic using mainly behavioral, social and cognitive processes. Leaders claimed their positioning through carrying out processes related to the different aspects of learning, mainly the cognitive, metacognitive, meta-emotional, social and linguistic aspects. Dominant emotions in the groups' learning were frustration, enjoyment and content.*

Keywords: Discursive framework, positioning, emotions, middle-achieving students.

Introduction

The affective aspect is a growing area in educational research due to its relationships with other aspects of students' learning, especially the cognitive one. In the present paper, we study middle-achieving students' emotions in relation to their positionings when they learned the quadratic function as a product of two linear functions using dynamic software. In a previous research, we examined students' positioning and emotions in one group learning geometry (Daher, Swidan & Shahbari, 2015), where the group consisted of two high-achieving students and one middle-achieving student. Following that study, we wondered how emotions and positioning are experienced by groups consisting of only middle achieving students. This paper intends to examine this issue, using the discursive framework (Evans, Morgan, & Tsatsaroni, 2006).

Emotions in mathematics education

Polya (1957) addressed the necessity to consider emotions as they influence the problem solving process. Later, especially in the 1980's, researchers considered affect as a significant component of students' mathematical problem solving (e.g., McLeod, 1988; Schöenfeld, 1985). Emotion is one of the fundamental elements of the affective aspect (Hannula, 2004). Hannula (2004) describes emotions as connected to personal goals. Furthermore, Emotions, when managed appropriately, become a potential tool for effective thinking rather than a disturbance to this thinking (Antognazza, Di Martino, Pellandini, & Sbaragli, 2015; Salovey & Mayer, 1990). In the present research, we intend to study, by using the discursive emotions and positioning framework, middle-achieving students' emotions when they utilize technology to study Algebra.

The discursive positioning framework for studying students' emotions

Discursive positioning is a conversational phenomenon through which individuals position themselves or others through actions (Davies & Harré, 1990; Skog, 2014). Evans, Morgan and Tsatsaroni (2006) suggest the discursive positioning framework for studying students' emotions. This framework assumes that meaning making occurs in social practices, using semiotic resources. Social practices have an emotional dimension that helps maintain social identity. Moreover, empirical data in this framework is seen as text, the analysis of which demands attention to its context(s). This analysis entails a combination of structural and textual phases that each informs the other. The structural analysis considers the positions available to or claimed by the participants. Positions are associated with power in relation to others, as well as with differing values within the discourse, which creates spaces within which emotion may arise. Usually, there is more than one available position for a participant, either within a single discourse or several competing discourses. Evans, Morgan and Tsatsaroni (2006) describe the positionings taken care of in the structural analysis: Helper and seeker of help, collaborator and solitary worker, director of activity and follower of directions, evaluator and evaluated, insider and outsider.

The textual analysis considers the exchange of meanings. This phase has two functions (Evans, 2006): (a) showing how positionings in social interactions are actually taken up by the participants, and (b) providing indicators of emotional experience. The textual analysis has two stages. In the first stage, the focus is to identify the interpersonal aspects of the text that establish the positions of the participants. Indicators at this stage include reference to self and others, reference to valued statuses (e.g. claiming understanding or correctness), modality (indicating degrees of un/certainty), hidden agency (e.g., passive voice) or repetition. For example, leadership is indicated by demonstrating knowledge or meta-emotional behavior (e.g., trying to change the negative emotions of the group members). The collaborator position is indicated by the activity of a group member, as answering questions or doing actions in response to events occurring during the group learning.

The second stage of the textual analysis attends to (a) indicators of emotional experience that include: direct verbal expression (e.g., 'I feel anxious'), use of particular metaphors (e.g. claiming to be 'coasting'), emphasis by words, gesture, intonation, or repetition, body language (e.g., facial expression); (b) indicators suggested by psychoanalytic theory, as indicators of defenses against strong emotions like anxiety, or conflicts between positionings (as 'Freudian slips'), surprising error in problem solving, behaving strangely (as laughing nervously), denial (e.g., of anxiety).

Research rationale and goals

In spite of mathematics education researchers' acknowledgement of the role of affective aspects in mathematical education in general and mathematical problem solving in particular, research related to this aspect is still not widespread (Antognazza et al., 2015). We intend to study emotions in problem solving using the discursive framework developed by Evans, Morgan and Tsatsaroni (2006). In more detail, we intend to analyze the positionings taken by ninth grade students and their related emotions when learning in groups, with the help of GeoGebra, the quadratic function as a product of two linear functions. Doing so, we introduce to the discursive framework the different

aspects of learning; as the meta-cognitive and meta-emotional aspects. This will shed more light on the factors that influence middle-achieving students' experiencing of positioning and emotions.

Research question

- How are positionings taken up by middle-achieving ninth grade students, working in a group to learn the quadratic function as a product of two linear functions, in the presence of technology?
- How are middle-achieving ninth grade students' emotions associated with the positionings that they claim, when learning with technology the quadratic function as a product of two linear functions?

Methodology

Research setting and participants

In a previous research (Daher et al., 2015); we analyzed the affective aspect of one group's learning of mathematics, where the members were both high and middle achieving students. We wondered how the affective aspect would be affected in just middle achieving or high achieving groups of students. In the present research, we analyze this aspect in three groups of grade 9 middle-achieving students (ages between 14 and 15 years). One group consisted of three female students (Sana, Amal, Asil), and two consisted of two female students and one male student each (Fairouz, Noura, Salim) and (Alaa, Siham, Amin). All the participating students had not worked with GeoGebra before, and they were introduced to it in two hours' session before learning the quadratic function topic. Furthermore, the students had learned some issues in the topic of the quadratic function (the function's maximum or minimum, the vertex of the function and the domain of increasing/decreasing), but not the quadratic function as a product of two linear functions. The third author taught the three groups in a middle school in Israel.

Data collecting and analyzing tools

We collected our data using observations of the learning of the three groups. We also conducted interviews with their members. Every group's learning was videoed and at the end of each lesson, the three students in each group were interviewed individually regarding their positionings and emotions during learning. We analyzed the two types of collected data using the discursive analysis framework presented above. Moreover, we combined the analyses of the data collected by the two tools (observations and interviews). The findings section in this paper sheds light on this method.

Learning material

The three groups of ninth grade students worked with a sequence of activities; all related to the quadratic function as a product of two linear functions. Following is an example of these activities.

In the same coordinate system, we want to draw the three functions: $y=x$, $y=x+2$ and $y=x(x+2)$.

- What are the algebraic characteristics of the linear function: $y=x$?
- What are the graphical characteristics of the linear function: $y=x$?
- What are the algebraic characteristics of the linear function: $y=x+2$?

- What are the graphical characteristics of the linear function: y=x+2?
- What are the algebraic characteristics of the linear function: y=x(x+2)?
- What are the graphical characteristics of the linear function: y=x(x+2)?
- What are the similarities and the differences between the characteristics of the two above linear functions and the characteristics of the quadratic function?

Note: Algebraic characteristics are related to the parameters of an equation, while the graphical characteristics are related to the intersection points with the axes, increasing or decreasing of a function, etc.

Findings

The present research aimed at characterizing middle-achievement students' positioning and emotions when learning algebra with technology. Doing so, we found that mainly the students had the leader and collaborator positionings during a lesson. We will describe how the students in the middle-achieving groups claimed each of the positionings and experienced their emotions and/or reported them in each positioning. Doing so, we will address the following aspects of learning that the positioning is related to: behavioral, cognitive, meta-cognitive, social and linguistic. The emotional aspect of learning will be considered in light of the taken positioning.

Collaborator's functioning

The middle-achieving groups utilized collaboration to learn the new mathematical ideas. This is exhibited in that generally the members of each of the participating groups claimed the collaborator positioning to pursue, with the help of the mathematical software, their learning of the quadratic function. This claiming resulted in making the group's members learn enthusiastically to understand the appropriate mathematical relations. This resulted in the group's members enjoying the activity and being content when arriving at its solution. Thus the collaborator's positioning helped make the students' emotions concerning their learning experiences positive ones.

To claim the collaborator positioning, the group members were involved with behavioral processes (working with GeoGebra), social processes (group discussions), as well as cognitive processes (mathematical reasoning). These three types of processes, not only helped the group members claim the collaborator positioning, but at the same time, supported their attempts, as described above, to perceive the new mathematical ideas. In the interview, the students associated their behavioral processes with positive and negative emotions: enjoyment of their work when the software helped them solve the mathematical problem, and frustration when finding difficulty to operate the software. Excerpt 1 shows this claiming the collaborator's positioning.

A1	Sana	We need to find the intersection with x for the three functions.
A2	Amal	What's the first function?
A3	Asil	3x-2
A4	Sana	[drew the first function in GeoGebra]
A5	Amal	What's the second function?
A6	Asil	2x+3

A7	Sana	[drew the second function in GeoGebra] Let's find the intersection points with x.

Excerpt 1: claiming the collaborator positioning

Excerpt 1 shows the claiming of the collaborator's positioning as connected with the behavioral aspect of the group's learning. This aspect is expressed by the students' action with the Geogebra software (A4, A7). However, this positioning also involves the meta-cognitive aspect. The utterances of Sana (A1, A7) are concerned with regulating the processes of the problem solution.

Leader's functioning

The leader's positioning in the middle-achieving groups was claimed by directing the learning of the group, as well as to advance this learning towards the solution of the mathematical problems and the sharing of the new mathematical ideas. Moreover, leaders in the middle-achieving groups claimed their positioning through carrying out different types of processes, mainly cognitive, metacognitive, meta-emotional, social and linguistic processes. Below, we elaborate on these processes.

The group leader's cognitive functioning was actualized through demonstrating knowledge during carrying out the mathematical activity. For example, Fairouz, a leader in one middle achieving group, argued that they only needed to know the intersection points of the two linear functions with the x axis in order to draw the resulting quadratic function.

The group leader's metacognitive functioning was actualized through asking questions during the group learning as means to decide upon the method of solving a problem. Moreover, the group leader's meta-emotional functioning was actualized through trying to change the negative mood of the group when encountering a difficulty. For example, Alaa, a leader in a middle-achieving group, tried to lessen the anxiety of group members by saying: "Don't worry. It's O.K. Sure we made a mistake. Let's read again our solution to find it".

The group leader's social functioning was actualized through answering other members' questions, asking questions and requesting actions from the group members to keep the group learning going. Regarding the linguistic aspect of the leader's functioning, the leaders in the middle-achieving groups used the first person plural pronoun to talk about the mathematical actions that they needed to perform, which showed them as collaborators with the other members of the group. This indeed happened in the middle-achieving groups but not numerously (See for example excerpt 2).

The leader's functioning resulted in different emotions, but generally speaking this functioning resulted in frustration, when unable to find a way for solving a mathematical problem, enjoyment during the successful solution process, and content when finally solving the activity.

Difficulties in claiming the positions of leaders and collaborators

The members of the middle-achieving groups, due to the lack of appropriate previous knowledge in the subject matter and sometimes in GeoGebra manipulation, encountered difficulties in claiming the positions of leaders and collaborators during the process of the mathematical problem solving. This led to their experiencing some negative emotions. Moreover, the members of the middle-achieving groups experienced calmness, anxiety and confusion in accordance with their leader.

Working with GeoGebra, the members of the middle-achieving groups encountered at the beginning difficulties related to working with a new technological tool, which could be related to the behavioral aspect of the group's learning. Excerpt 2 describes such a difficulty, where Salim, Noura and Fairouz wanted to draw the function (2x-9)(3x-4) in GeoGebra [B1], but found difficulty doing that due to not writing correctly the appropriate number of brackets [B2-B6].

B1	Fairouz	We should write 3x-4 multiplied by 2x-9. [Noura started to write the expressions]
B2	Salim	Perhaps the brackets can be put afterwards, wait Noura, wait, it keeps moving.
B3	Fairouz	Write it from the beginning.
B4	Salim	No, no [He takes the mouse from Noura who seemed annoyed by the act of Salim. Salim works on GeoGebra] O.K. Now write it again.
B5	Noura	[Noura puts her hand on her mouth with boredom]
B6	Fairouz	[Fairouz worked on GeoGebra, then she said with annoyance] something wrong with the brackets.
B7	Teacher	Don't get anxious. At the beginning, brackets are tricky. Everything will get O.K.

Excerpt 2: Students' difficulties in working with GeoGebra and related emotions

Excerpt 2 shows some of the difficulties encountered by the middle-achieving groups, as a result of their behavioral functioning; specifically when working with a technological tool. Fairouz, in the interview, said they felt out-of-control and thus frustrated not being able to draw from the beginning the graph of the function f(x)= (2x-9)(3x-4) in GeoGebra. Salim pointed at the teacher's interference as supporting them in getting back control over their work with GeoGebra, which made them satisfied with their work on the mathematical problem.

Encountering difficulties in learning the new topic, not only influenced students' positioning and emotions, but also colored the linguistic aspect of their learning, especially their use of pronouns. This is the case in excerpt 3, where the difficulty is related to simplifying an algebraic expression.

C1	Siham	We want to draw the quadratic function y=(2x-9)(-x-4).
C2	Amin	Multiply first the brackets.
C3	Alaa	(-x-4)(2x-9)= $-2x^2$+9x-8x+36, Now we compute +9-8.
C4	Amin	-17
C5	Siham	Minus 8 plus 9.
C6	Amin	Minus 17, plus .. [He seemed anxious, not sure of his computation]
C7	Alaa	What?
C8	Amin	Minus one or plus one
C9	Alaa	What?
C10	Siham	Plus 1.
C11	Alaa	Write $-2x^2$+x-36.

Excerpt 3: Having difficulty in simplifying an algebraic expression

Excerpt 3 shows that confronting difficulty constrained the group's sense of control and produced anxiety. In this situation, singular pronouns or no pronouns were used.

Discussion and conclusions

Research of students' emotions in mathematics learning is growing (e.g., Antognazza et al., 2015; Daher, 2011; Hannula, 2004). The present research aimed at characterizing grade 9 students' positioning and emotions when learning algebra with technology. The research findings indicate that to claim the collaborator positioning, members of the middle-achieving group were involved with behavioral processes (working with GeoGebra), social processes (class discussions), as well as cognitive processes (reasoning). These processes helped them reach their learning goal, thus resulting in positive emotions. It could be said that collaboration was associated mainly with positive emotions as enjoyment and content, though negative emotions as anxiety were experienced when having difficulty in solving the mathematical problem; i.e. in arriving at the learning goal.

To claim the leader positioning in a middle-achieving group, the member was involved with different learning processes, as demonstrating knowledge, which was also reported in Evans, Morgan and Tsatsaroni (2006), but their functioning was distinguished from the other group members by performing metacognitive and meta-emotional processes, as reported in Daher et al. (2015). These processes helped plan, monitor, evaluate and take decisions regarding the group learning, especially in time of difficulty in arriving at the learning goals. Thus, these processes helped maintain the leader positioning (Black, Soto & Spurlin, 2016), as they supported the leader in advancing the group learning.

In addition, the leader metacognitive functioning was actualized by asking questions as means to decide upon the method of solving a problem. This decision making could be looked at as a social process (Vroom & Jago, 1974) with the goal to advance the group learning. Moreover, it seems that critical thinking skills, actualized in decision making, were needed to claim the leader's positioning. Furthermore, the goal of the leader meta-emotional functioning was to change the negative mood of the group when encountering a difficulty, which motivated the members' work (Leithwood, Louis, Anderson & Wahlstrom, 2004). So, we argue that the leader positioning was claimed by paying attention to different aspects of the group learning, especially the metacognitive and meta-emotional aspects. This leader's functioning resulted in different emotions related to the difficulty and success in performing the mathematical task, which could be associated with Goldin's (2000) emotional pathway, where generally frustration preceded enjoyment and enjoyment preceded content. This emotional pathway included the three dominant emotions in the groups' learning, i.e. no singular emotion was dominant but the emotional pathway was thus. Furthermore, the group members' emotional experience was influenced by that of the leader, which could be related to the emotional contagion suggested by Hatfield, Cacioppo and Rapson (1993), where there is tendency to converge emotionally with others. We say this is especially true in group learning when the other is the leader.

Students encountered sometimes difficulties in learning the new topic. This encounter, not only influenced students' positioning and emotions, but also their linguistic use of pronouns. This was expressed in their use of singular pronouns or no pronoun at all when getting anxious for not being able to proceed with the carrying out of the activity.

Future studies are needed to compare the positioning and emotions of different achievement-groups in solving mathematical problems. This could be done utilizing the same theoretical framework used in the present research.

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