From theory through collaboration into practice: designing a problem solving curriculum for grade 6 students

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Teachers are continuously confronted with instructional endorsements, such as the inclusion of problem solving in school mathematics. However, adoption of problem solving is still not a reality. One reason for it is lack of practical teaching materials based on research findings to achieve the goals stated in the standards. In the context of this reform agenda, collaborative work between educational researchers and practitioners in a real setting working on issues of everyday practice is crucial in order to overcome the gap between theory and practice. In this paper, I focus on such theory based problem solving curriculum for grade 6 students that were developed using design-based research. At the end, I discuss factors inhibiting the implementation of the curriculum.

Keywords: Word problems, material development, mathematics activities.

Introduction

The (inter-)national educational standards (e.g., KMK, 2003; NCTM, 2000) have strongly endorsed the inclusion of problem solving in school mathematics. Empirical studies, however, portray a different picture: Students are often unable to solve problem tasks (e.g., Schoenfeld, 1985) and teachers lack practical teaching materials based on research findings to achieve the goals stated in the standards (Kuzle & Gebel, 2016). In the context of this reform agenda, the development of materials for students and teachers is of great importance for overcoming the gap between theory and practice (Jahn, 2014). Nowadays, quality analyses in the German school system evaluate the compliance of teaching practices with the educational state standards. As reported in Kuzle and Gebel (2016), one such quality analysis was conducted in one urban school. The results in the area of problem solving painted a rather poor picture: problem solving tasks were rarely introduced. When this was the case, they were primarily done by the teachers; mostly routine tasks dominated the lessons, and problem solving strategies were explicitly applied in one third of examples only. The school recognized this deficit and set as a goal promoting problem solving instruction centered around curriculum material developed through collaborative work between educational researchers and practitioners using design-based research (DBR) as a research paradigm.

In this paper I report on a small part of the SymPa[1]-research project (Systematical and material based development of problem solving competence) focusing on collaboration between practitioners and researchers with the goal of developing a problem solving curriculum for grade 6 students. The guiding question is: What factors inhibit implementation of research-based problem solving curriculum in practice? In the following sections I outline relevant theoretical and methodological underpinnings used to design a problem solving curriculum, before showing how

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[1] SymPa stands for „**<u>Sy</u>**stematischer und **<u>m</u>**aterialgestützter **<u>P</u>**roblemlösekompetenz**<u>a</u>**ufbau". Inga Gebel (researcher) and Christian Conradi (practitioner) initiated and participated in the project likewise.

these got implemented, and report on its evaluation (initial DBR-cycle). As a result of evaluation, I discuss the curricular redesign that might allow more effective implementation in practice.

Theoretical foundation guiding the design process

Plethora of research on problem solving undergoing since the 1970s identified several pivotal areas for a problem solving curriculum. I outline here only a small portion of this research that was crucial for the project based on German standards' conception of problem solving (KMK, 2003).

Problem solving competence

Problem solving competence relates to *cognitive* (here heuristic), *motivational* and *volitional* knowledge, skills and actions of an individual required for independent and effective dealing with mathematical problems (Bruder, 2002; Kuzle & Gebel, 2016). Accordingly, students should a) learn approaches (heuristics) for solving mathematical problems and how to apply them appropriately in a given situation, b) develop reflectivity for own actions, and c) develop willingness to work hard (cf. Bruder, 2002; KMK, 2003). As problem solving competence encompasses so many different facets, problem solving curriculum should account for the following research areas: (a) teaching approaches and concepts on problem solving, (b) theories of self-regulated learning and self-regulation in problem solving, and (c) theories of motivation which are outlined below.

Teaching approaches and concepts on problem solving in combination with self-regulation

There are at least seven practices for problem solving curriculum that researchers (e.g., Kilpatrick, 1985; Pólya, 1945/1973; Schoenfeld, 1985) have claimed to be important for helping students grow in their problem solving ability: (1) osmosis (give lots of problems), (2) give "good" problems, (3) memorization (teach specific or general heuristic strategies (heurisms)), (4) imitation (model problem solving), (5) cooperation (limit teacher input by having students work in small groups), (6) reflection (promote metacognition by asking metacognitive questions or encouraging students to be reflective), and (7) highlight multiple solutions. In the recent years, Bruder (2002) developed a problem solving teaching concept focusing around Lompscher's (1975) idea of "flexibility of thought". *Flexibility of thought* is expressed by one' ability to

1. reduce a problem to its essentials or to visualize it by using visual and structuring aids, such as informative figures, tables, solution graphs or equations (*reduction*).
2. reverse trains of thought or reproduce these in reverse, such as by working backwards (*reversibility*).
3. simultaneously mind several aspects of a given problem or to easily recognize any dependences and vary them in a targeted manner (e.g., by composing and decomposing geometric figures and objects, by working systematically) (*minding of aspects*).
4. change assumptions, criteria or aspects in order to find a solution, such as by working forwards and backwards simultaneously or by analyzing different cases. Such ability prevents "getting stuck" and allows new approaches and insights (*change of aspects*).
5. transfer an acquired procedure into another context or to very one by using analogies for instance (*transferring*).

These typical manifestations of mental agility can be related to analyses of heuristic approaches by Pólya (1945/1973). Untrained problem solvers, however, are often unable to access the above outlined flexibility qualities consciously. Moreover, not only the knowledge of different heuristics (flexibility of thought) is needed when problem solving, but also self-regulatory abilities which evolve gradually through a 5-phase model (Zimmerman, 2002). Problem solving can be trained by learning heuristics corresponding to these aspects of *intellectual flexibility* in combination with *self-regulation* (Zimmerman, 2002), which according to Bruder (2002) and (Bruder & Collet, 2011) develops through the following five-phase concept:

1. *Intuitive familiarization*: This phase builds on Pólya's (1945/1973) model, in which a teacher serves as a role model when introducing a problem to students. Thus, the teacher moderates behaviors typical for the problem by engaging in self-questioning pertaining to different phases of the problem solving process (before, during, and after). For example questions such as, "What is the problem asking for?" "What information am I given?" "Is there anything I don't understand?" "Am I headed in the right direction?" may help guide the students (Kuzle & Bruder, 2016). At this point the heurism in focus is not specified.
2. *Explicit strategy acquisition*: During this phase the students get explicitly introduced to the heurism in focus on the basis of a reflection from the first phase. Here the particularities of the heurism get discussed and it gets a name (exemplification). Here prototypical problems get used for introducing a heurism in focus, so that the students can more easily recognize its main ideas and more easily remember their specific steps for future problem solving.
3. *Productive practice phase*: During this phase the students practice solving the problems using the heurism in focus. Here is important that the problems do not reproduce type problems, bur rather expand the possibilities from the first two phases. In addition, differentiation should be a guiding concept during this phase, so that students can choose at what cognitive level they want to work and adapt the observed learning behavior.
4. *Context expansion*: In this phase the students should practice the use of heurism in focus independent of a mathematical context. In that way, the students learn to flexibly, unconsciously and independently of a context use the heurism in focus.
5. *Awareness of own problem solving model*: The aim of the teaching concept is that the problem solving model of the students gets expanded, so that they are in a position to solve problems better using different heurisms. Awareness of own problem solving model can be induced by having students reflect and document their problem solving process.

Lastly, students' willingness to work hard is a major factor for the successful problem solving process. Without an effort from the learners, there will be no successful learning. For that reason, the criteria such as, understandable and clear problem, age-appropriate choice of context, and visible competence growth (Bruder, 2002) are crucial when designing problem solving curriculum.

To summarize: the problem solving curriculum was developed around the operationalization of the problem solving competence. This included the teaching concept of problem solving by Bruder (2002) in combination with Zimmerman's (2002) self-regulation model taking into consideration the criteria for motivating tasks.

Curriculum development

The problem solving curriculum was developed in collaboration between the two researchers and one practitioner (teacher from the project school). More concretely, the researcher team developed the curriculum based on the outlined theories and school's contextual factors (see below), which were discussed up-front. Curriculum materials (e.g., problems, textual parts, figures, colors) were either separately developed by the researcher team and afterwards discussed with the teacher or the entire team met together and developed it. The final decision about the problem solving curriculum (e.g., content, problems) was met by the teacher.

Enactment

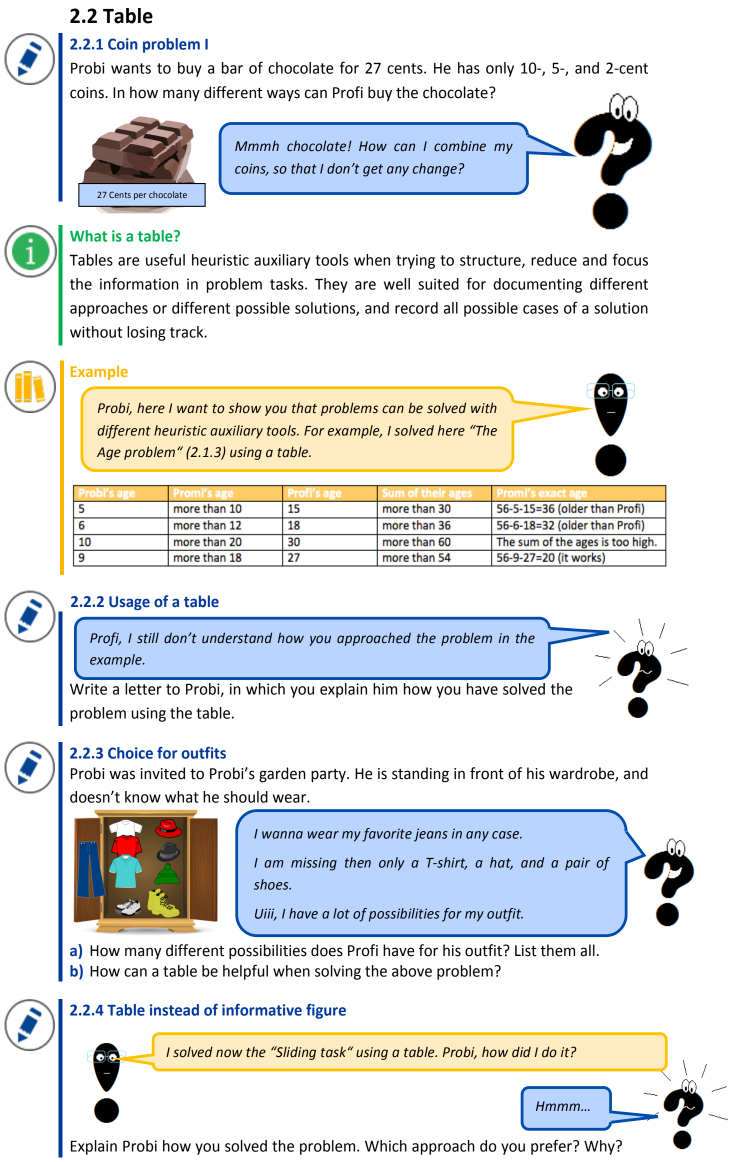
For the design of curriculum contextual factors played a great role, in which theoretical ideas had to be operationalized. Students of 6th grade were chosen to participate in the project lasting one school quarter (ca. 16 lessons, 1 lesson = 45 min). The implementation of the curriculum took place during two parallel phases (see Table 1). During the first DBR-cycle 13 students participated. Teacher A initiated the project, had previous experience in problem solving (e.g., attended professional development courses on problem solving, read literature on it, and implemented problem solving tasks occasionally in his teaching practices. The second DBR-cycle started parallel to the first DBR-cycle, and was led by another mathematics teacher. In total 12 students participated. Teacher B had practically no experience with problem solving or teaching problem solving.

1st DBR-cycle	2nd DBR-cycle
every 14 days (8 meetings), Fridays, double period, teacher A	weekly (17 meetings), Mondays and Tuesdays, single periods, teacher B

Table 1: Parallel enactment cycles

With respect to heurisms, focus laid on those heurisms prescribed by the school's own curriculum, namely heuristic auxiliary tools (informative figure, table, solution graph), heuristic strategies (working systematically, working forwards, working backwards), and heuristic principles (composing and decomposing). Thus, all flexibility qualities were addressed. With respect to mathematical content, problems covered the content areas of 5th and 6th grade mathematics (operations with natural numbers and fractions, combinatorics, geometric and numeric patterns, measurement pertaining to 2- and 3-dimensional figures). Based on the project time frame, each heurism was covered within two lessons, but followed the above underlined problem solving concept. For one exemplarily operationalization with references to theoretical base see Figure 1.

During the implementation phase data collection took place on three different levels: student level, teacher level and classroom level. With respect to the student level, data from student textbooks (intermediate reflections, final reflection) and their workbooks (student productions) was collected. With respect to the teacher level, data from continuous communication with the teachers (e-mail, telephone calls), teacher textbook and semi-structured interview at the end of the project was collected. Concretely, continuous communication allowed the researcher team to support the teachers during the implementation phase with respect to pedagogical and/or methodological questions (e.g., discussion of different solutions, cooperative methods), by answering questions of content nature (e.g., questions about particular heurism), and through flexible and stepwise redesign of the curriculum after each lesson. Lastly, with respect to the classroom level, observations allowed for analysis of student-teacher interaction, and students' interaction with the curriculum.

Curriculum on the heuristic auxiliary tool of table	**Theoretical foundation**
	In the phase of intuitive familiarization, students solve a representative problem for the heurism in focus together with the teacher, who serves as a moderator. Here the imitation of teachers' behavior takes place through self-questioning. The problem represents the students' first encounter with the heurism in focus. In the phase of explicit strategy acquisition, the heurism in focus gets formally introduced through a short student-centered information text and an example. In what follows, at least three problems of different cognitive level are presented that serve as a productive practice phase. This allows for differentiation, where each student can solve as many problems as he or she can. In addition, problems from different mathematical content areas are covered, to allow for transfer (context expansion phase), which pertains to the fourth phase of the teaching concept. In addition, the heurisms are interrelated, so it is important that the students comprehend this. The last task allows students to make this connection by comparing the two heurisms and reflect on it.

2.2 Table

2.2.1 Coin problem I

Probi wants to buy a bar of chocolate for 27 cents. He has only 10-, 5-, and 2-cent coins. In how many different ways can Profi buy the chocolate?

What is a table?

Tables are useful heuristic auxiliary tools when trying to structure, reduce and focus the information in problem tasks. They are well suited for documenting different approaches or different possible solutions, and record all possible cases of a solution without losing track.

Example

Probi's age	Promi's age	Profi's age	Sum of their ages	Promi's exact age
5	more than 10	15	more than 30	56-5-15=36 (older than Profi)
6	more than 12	18	more than 36	56-6-18=32 (older than Profi)
10	more than 20	30	more than 60	The sum of the ages is too high.
9	more than 18	27	more than 54	56-9-27=20 (it works)

2.2.2 Usage of a table

Write a letter to Probi, in which you explain him how you have solved the problem using the table.

2.2.3 Choice for outfits

Probi was invited to Probi's garden party. He is standing in front of his wardrobe, and doesn't know what he should wear.

a) How many different possibilities does Profi have for his outfit? List them all.
b) How can a table be helpful when solving the above problem?

2.2.4 Table instead of informative figure

Explain Probi how you solved the problem. Which approach do you prefer? Why?

Figure 1: A sample page from the problem solving curriculum on the heuristic auxiliary tool of table

The problems focused on contexts that are motivating and appropriate for young students. In addition two figures were introduced to support students' willingness to work hard; they could identify with Probi (shape of a question), who asks questions and gets stuck. Profi (shape of an exclamation mark) offers then support to students, who illustrates a professional problem solver.

Evaluation

Qualitative-content analysis was used to analyze the collected qualitative data as outlined in Patton (2002). This method is particularly suitable for research activities, in which the knowledge is low and a study has more of an exploratory character. Thereby, the aim was to systematically analyze the qualitative data and produce a category system by focusing on factors inhibiting the implementation of the curriculum. The deductive analysis was performed based on the theoretical foundation, which was then refined in the inductive analysis by emerging issues and additional codes. The situations were interpreted as inhibiting when they allowed for a limited implementation of the curriculum only as reported explicitly by teachers and students and/or was observed by reviewing the collected data. As a result four categories were produced (see Table 2). The category system was then used to interpret the results of the study with respect to the research question. All data got analyzed by the two researchers independently.

Category	**Description**	**Subcategory**
Language	All language-based comments that influenced the understanding of the problem were assigned to the language category.	Solution graph terminology Figure names Problem enumeration
Level of performance	Barriers influencing the level of performance during problem solving process (e.g., content barriers, subjective barriers) were assigned to the level of performance category.	Motivation and differentiation Curricular difficulties Increased level of performance
Learning pedagogies	The evaluations of the curriculum in terms of the didactic ideas about learning how to solve problems are listed in the learning pedagogies category.	Communication ability Reflective ability
School and personal influences	Any feedback that aimed at external factors influencing the implementation was assigned to the school and personal influences category.	Teacher attitude Professionalism Organization

Table 2: Four content categories inhibiting implementation of the problem solving curriculum

Table 2 shows that also school and personal influences, which were not part of the design process, influence the extent to which the curriculum gets implemented. Thus, teachers also inhibit successful implementation of the curriculum, despite being part of the design process. I focus on this category by giving different subcategory examples since this factor was most surprising.

Figure 2 shows student's work solving one problem where the heuristic auxiliary tool of informative figure was to be used. Instead of representing a graphical illustration of the situation to be resolved from which the solution can be "read" (informative figure), it represents a sorting of information. Hence, the student work is a reflection of teacher's B lack of knowledge of problem solving, despite this problem being discussed in the teacher manual, which she glanced only once (professionalism). This was seen in all students' notes. Moreover, the analysis of students' workbooks revealed that teacher B avoided introducing problem solving terminology and allowed students to solve problems as they wished (teacher attitude). She explained that she did not want to burden them with formal terminology and constraint their problem solving process. Also teacher B criticized heavily the name of the figures in the curriculum (Profi, Probi), which was reflected in students' final reflection, where they urged for the change of figure names. Thus, teacher's negative attitude about a design element transferred negatively onto the students. Such behavior was not observed with the students led by teacher A, who questioned the chosen names, but never criticized them during his instruction. Teacher B in comparison to teacher A was not part of the design team, but was assigned to implement the curriculum due to organizational school context. The case of teacher B shows that negative attitude and lack of professional behavior inhibited successful implementation of the curriculum. Hence, teacher as an inhibiting factor should not be neglected during the design and implementation process.

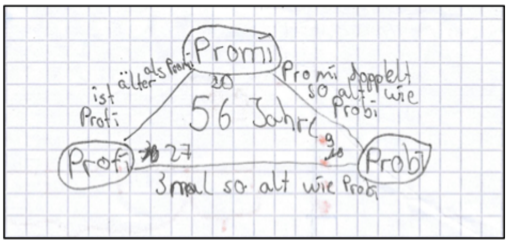


Figure 2: Teacher's professionalism reflected in a sample page from one student's workbook

Conclusion

Problem solving must gain more importance in school mathematics. Although several teaching concepts and practices are known, these get rarely implemented. Moreover, curriculum based on existing and empirically tested problem solving pedagogies is non-existent. To overcome this gap SymPa-project was grounded. Teachers participating in the project reported improvement in students' problem solving competences with respect to deliberate and mindful use of different heurisms when problem solving. In addition, the teachers not participating in the project, reported students using these heurisms in regular mathematics classes. Hence, it was possible to develop curriculum that met the local demands with the aim of supporting a systematical development of problem solving competence. However, different objective and subjective factors inhibited full-implementation of the curriculum. With respect to the former (language, level of performance, learning pedagogies), changes done in the re-design phase of the DBR-cycle will shed light to which extent these were enough for successful implementation in the upcoming phases of enactment. With respect to the latter (school and personal influences), it became clear that the curriculum alone does not guarantee the implementation of the teaching concept. Substantial knowledge base of the content and pedagogical ideas seem necessary to teach in accordance with the theoretical foundation. Confidence and experience in teaching problem solving played a crucial role likewise. Likewise school organizational factors should not be ignored. Since the teachers were assigned to teach the problem solving lessons and received no compensation for the participating in the project, a lack of motivation may develop, which influences the willingness to teach, the lesson quality and with it the students' acceptance of the curriculum.

In this paper I demonstrated that DBR-paradigm allows creating novel teaching environments in which theory and practice were not detached from one another, but rather complemented each other. Here the efforts were made to design, use and do research on problem solving curriculum in a real setting. This promoted adoption of the innovation – problem solving curriculum – which became an official part of that school's curriculum. Moreover, close collaboration during the design and enactment phase, and the re-design of the materials by the researchers as a result of teachers' feedback allowed them to develop a sense of ownership for the designed curriculum. Future work should use similar methodologies to ensure implementation and adoption of research into practice.

References

Bruder, R. (2002). Lernen geeignete Fragen zu stellen. Heuristik im Mathematikunterricht. *mathematik lehren, 115*, 4–8.

Bruder, R., & Collet, C. (2011). *Problemlösen lernen im Mathematikunterricht*. Berlin: Cornelsen Scriptor.

Jahn, D. (2014). Durch das praktische Gestalten von didaktischen Designs nützliche Erkenntnisse gewinnen: Eine Einführung in die Gestaltungsforschung. *Wirtschaft und Erziehung, 66*, 3–15.

Kilpatrick, J. (1985). A retrospective account of the past twenty-five years of research on teaching mathematical problem solving. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (pp. 1–16). Hillsdale, NJ: Erlbaum.

Kultusministerkonferenz (2003). *Bildungsstandards im Fach Mathematik für den mittleren Schulabschluss*. Bonn: KMK.

Kuzle, A., & Bruder, R. (2016). Probleme lösen lernen im Themenfeld Geometrie. *mathematik lehren, 196,* 2–8.

Kuzle, A., & Gebel, I. (2016). Development of materials for problem solving instruction in the context of lessons for promoting and improving specific mathematical competences using design based research. In T. Fritzlar, D. Assmuss, K. Bräuning, A. Kuzle, & B. Rott (Eds.), *Problem solving in mathematics education. Proceedings of the 2015 Joint Conference of ProMath and the GDM Working Group on Problem Solving* (pp. 159–172). Münster: WTM-Verlag.

Lompscher, J. (1975). *Theoretische und experimentelle Untersuchungen zur Entwicklung geistiger Fähigkeiten*. Berlin: Volk und Wissen.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: Author.

Patton, M. Q. (2002). *Qualitative research and evaluation methods*. Thousand Oaks, CA: Sage.

Pólya, G. (1973). *How to solve it: A new aspect of mathematical method*. Princeton, NJ: Princeton University Press. (Original work published 1945).

Schoenfeld, A. H. (1985). *Mathematical problem solving*. Orlando, FL: Academic Press.

Zimmerman, B. J. (2002). Becoming a self-regulated learner: An overview. *Theory Into Practice, 41*(2), 64–70.